ASE 211 Homework 8 Solution

1. Write an m-file which takes as input the data points (x_i, y_i) $i = 1, \ldots, n$ and constructs the spline matrix and right hand side, and solves for the vector of second derivatives.

```
function [S]=splinemat(x,y,n)
%  function solves for the vector of second derivatives
%  of a spline function
%
for i=1:n-1
   h(i)=x(i+1)-x(i);
end
A(1,1)=1;
A(n,n)=1;
b(1)=0;
b(n)=0;
for i=2:n-1
   A(i,i)=2*(h(i-1)+h(i));
   b(i)=6*((y(i+1)-y(i))/h(i)-(y(i)-y(i-1))/h(i-1));
end
for i=2:n-2
   A(i,i+1)=h(i);
   A(i+1,i)=h(i);
end
b=b'
S=A\b;
```

2. Write another m-file which takes as input the vector of second derivatives, and the data points, and computes the coefficients of the spline function.

```
function [a,b,c,d]=evalcoef(x,y,n,S)
for i=1:n-1
   h(i)=x(i+1)-x(i);
end
for i=1:n-1
   b(i)=S(i)/2;
   a(i)=(S(i+1)-S(i))/(6*h(i));
   c(i)=(y(i+1)-y(i))/h(i)-(S(i+1)-S(i))/6*h(i)-S(i)/2*h(i);
```

```
   d(i)=y(i);
end
```

3. Test your program on the data in problem A8.7. Plot your spline function, and find the value of the spline at $T = 50$.

```
function plot_spline(a,b,c,d,n,x,y)
% function which plots a spline given its coefficients
% a(i),b(i),c(i),d(i), i=1 to n-1
% and the data points x(i), i=1 to n
%
aa=x(1);
bb=x(n);
h=(bb-aa)/100;
for i=1:100
   xx(i)=aa+i*h;
   for j=1:n-1
     if ((x(j) <= xx(i)) & (xx(i) <= x(j+1)))
       yy(i)=a(j)*(xx(i)-x(j))^3+b(j)*(xx(i)-x(j))^2+c(j)*(xx(i)-x(j))+d(j);
     end
   end
end
plot(xx,yy,x,y,'+')
%  evaluate the spline at t=50


>> x=[40; 48; 56; 64; 72]

x =

    40
    48
    56
    64
    72

>> y=[55.3; 83.7; 123.8; 179.2; 254.5]

y =
```

```
   55.3000
   83.7000
  123.8000
  179.2000
  254.5000

>> n=5;
>> S=splinemat(x,y,n)

A =

     1     0     0     0     0
     0    32     8     0     0
     0     8    32     8     0
     0     0     8    32     0
     0     0     0     0     1


b =

         0
    8.7750
   11.4750
   14.9250
         0


S =

         0
    0.2247
    0.1982
    0.4169
         0

>> [a,b,c,d]=evalcoef(x,y,n,S)

a =
```

```
    0.0047   -0.0006    0.0046   -0.0087

b =

         0    0.1123    0.0991    0.2084

c =

    3.2504    4.1491    5.8406    8.3009

d =

   55.3000   83.7000  123.8000  179.2000

>> plot_spline(a,b,c,d,n,x,y)
 the value at t=50

yt =

   92.4431
```

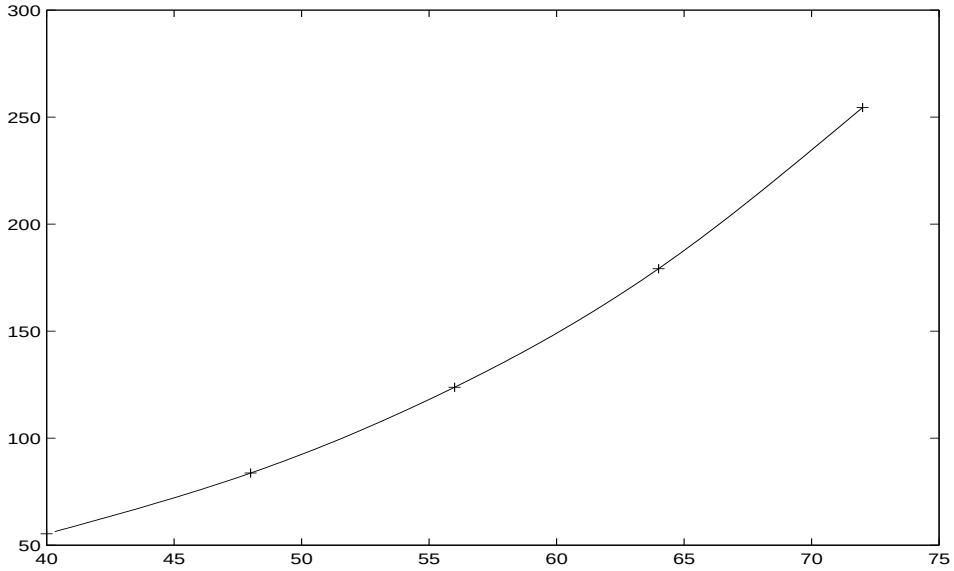

Figure 1: Plot of spline