ASE 211 Homework 11 Solution

Due: 12:00 noon, Friday, Nov 17.

1. Write a Matlab m-file which implements the composite Simpsons's Rule. Your input to the m-file should be the limits of integration a, b and the number of subintervals N. Remember that N has to be even.

```
function y=simp(a,b,N)
% M-file performs Simpson's rule
% note: N must be even for this to work properly
%
h=(b-a)/N;
x=a-h;
y=0;
for i=2:2:N
   x=x+2*h;
   y=y+4*f(x);
end
x=a;
for i=3:2:N-1
   x=x+2*h;
   y=y+2*f(x);
end
y=y+f(a)+f(b);
y=y*h/3;
```

Test your m-file on the integral

$$\int_0^1 x^3 dx = .25.$$

It should get the correct answer for any value of N.

Matlab diary:

```
N=2;
a=0;
b=1;
y=simp(a,b,N)

y =
```

```
    0.2500
```

Next, test your m-file on the integral

$$\int_0^1 x^5 dx = 1/6,$$

and show that as you double $N = 2, 4, 8$, the error goes to zero like h^4.

Matlab diary:

```
N=2;
a=0;
b=1;
for i=1:3
err(i)=abs(simp(a,b,N)-1/6)
if (i~=1) ratio=err(i-1)/err(i)
end
N=N*2;
end

ratio =

   16.0000


err =

  1.0e-005 *

    0.5086    0.0318    0.0020


ratio =

   16.0000
```

Since the error ratio is 16, this indicates h^4 convergence.

Test your m-file on the following integral:

$$\int_0^4 x^{2.8} e^{-2x^{.1}} dx$$

and by taking a sufficient number of subintervals N, determine the integral to within four digits of accuracy.

```
a=0;
b=4;
N=4;
y=simp(a,b,N)

y =

    5.4504

N=8;
y=simp(a,b,N)

y =

    5.4526

N=16;
y=simp(a,b,N)

y =

    5.4528

N=32;
y=simp(a,b,N)

y =

    5.4528
```

The answer is 5.4528.

2. Write a Matlab m-file which implements the composite 2-point Gauss quadrature formula. Test this m-file on all of the integrals in problem 1. In particular, the error should also go to zero like h^4.

For $f(x) = x^3$:

```
a=0;
```

```
b=1;
N=1;
y=gauss2pt(a,b,N)

y =

    0.2500
```

For $f(x) = x^5$:

```
a=0;
b=1;
N=1;
for i=1:3
err(i)=abs(gauss2pt(a,b,N)-1/6)
if (i~=1) ratio=err(i-1)/err(i)
end
N=2*N;
end


ratio =

   16.0000


err =

    0.0139    0.0009    0.0001


ratio =

   16.0000
```

For $f(x) = x^{2.8}e^{-2x^{.1}}$:

```
a=0;
b=4;
```

```
N=4;
y=gauss2pt(a,b,N)

y =

    5.4530

N=8;
y=gauss2pt(a,b,N)

y =

    5.4529
N=16;
y=gauss2pt(a,b,N)

y =

    5.4528

N=32;
y=gauss2pt(a,b,N)

y =

    5.4528
```