ASE 211 Homework 10 Solution

1. Consider the function $f(x) = 3x^{2.5}\cos(x^2)$. Compute the derivative of f at $x = \sqrt{\pi}$ analytically. Compute the forward difference approximation to $f'(\sqrt{\pi})$ for $h = .1$, .05 and .025, and show that the error is going to zero like h. Compute the central difference approximation and show that the error is going to zero like h^2.

$$\begin{aligned} f'(x) &= 7.5x^{1.5}\cos(x^2) - 6x^{3.5}\sin(x^2) \\ f'(\sqrt{\pi}) &= -17.6980 \end{aligned}$$

Let $x = \sqrt{\pi}$, $d_h = (f(x+h) - f(x))/h$, and $e_h = |f'(x) - d_h|$:

h	d_h	e_h	$e_h/e_{h/2}$
.1	-8.995	8.6995	–
.05	-13.7400	3.9580	2.1980
.025	-15.8169	1.8811	2.1041

Since the ratio of errors is approaching 2 as h is divided by 2, the rate of convergence is $\mathcal{O}(h)$.

Let $c_h = (f(x+h) - f(x-h))/(2h)$, $e_h = |f'(x) - c_h|$:

h	c_h	e_h	$e_h/e_{h/2}$
.1	-16.1652	1.5328	–
.05	-17.3116	.3864	3.97
.025	-17.6012	.0968	3.99

Since the ratio of errors is approaching 4 as h is divided by 2, the rate of convergence is $\mathcal{O}(h^2)$.

2. A rocket flying straight upward during launch sends back the following velocity data:

t (sec)	v(t) (m/s)
.5	5.1
1.5	8.9
2.8	16.9
3.1	18.1
4.0	27.9
5.5	34.3
6.2	35.1
6.5	35.5
7.0	38.1
8.2	39.2

Using your spline code from previous assignments, plot the acceleration function $a(t) = v'(t)$ from $t = 0$ to $t = 8.2$, obtained by differentiating the spline interpolant of the velocity data.

This involves creating the spline as we did before, then modifying the function which plots the spline to plot the derivative of the spline as well.

```
function plot_spline(a,b,c,d,n,x,y)
% function which plots a spline given its coefficients
% a(i),b(i),c(i),d(i), i=1 to n-1
% and the data points x(i), i=1 to n
%
aa=x(1);
bb=x(n);
h=(bb-aa)/100;
for i=1:100
   xx(i)=aa+i*h;
   for j=1:n-1
     if ((x(j) <= xx(i)) & (xx(i) <= x(j+1)))
       yy(i)=a(j)*(xx(i)-x(j))^3+b(j)*(xx(i)-x(j))^2+c(j)*(xx(i)-x(j))+d(j);
       yd(i)=3*a(j)*(xx(i)-x(j))^2+2*b(j)*(xx(i)-x(j))+c(j);
     end
   end
end
plot(xx,yy,'--',x,y,'+',xx,yd)
```

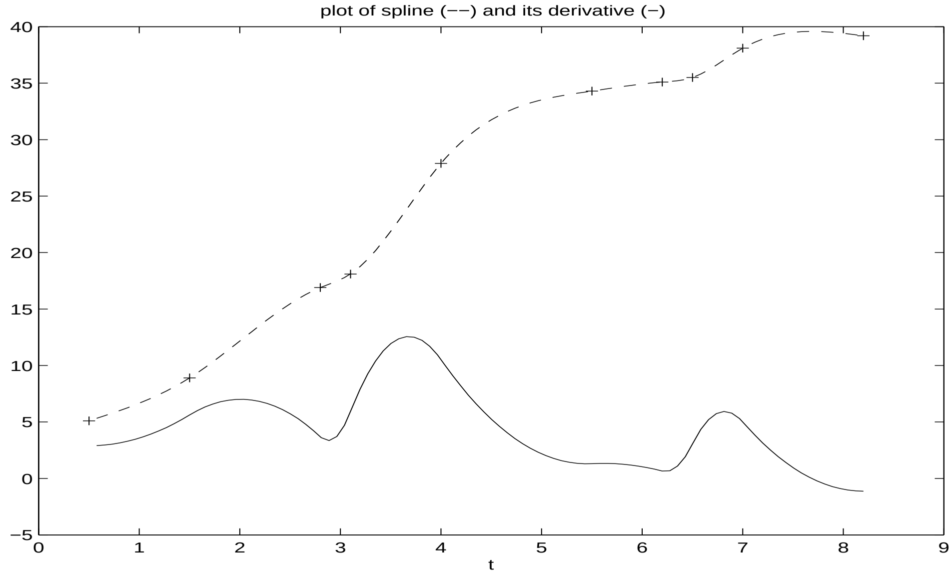

Figure 1: Plot of spline (velocity) and its derivative (acceleration). The derivative is the solid line.