ASE 211 Homework 12

Due: Friday, Dec. 1 at noon.

An airplane's velocity v as a function of position x along its path is determined to satisfy

$$\frac{dv}{dx} = v'(x) = (x-100)^2 v^{.25}/1900 = f(x,v), \quad x > 0,$$
$$v(0) = 50 \text{ km/h}$$

Write two matlab functions, one which approximates v using Euler's method, and one which uses the second-order Runge-Kutta (RK2) method. For the RK2 method, find a step-size h for which the approximation V_h at $x = 100$ km satisfies

$$|V_{2h}(100) - V_h(100)| < .25. \tag{1}$$

Tell me what h is, and plot V_h from $x = 0$ to $x = 100$. (Hint: start with a reasonable h, say $h = 1$, and divide h by 2 until the condition above is satisfied.)

Consider the value $V_h(100)$ to be the "exact" solution $v(100)$. For the Euler method, find a step-size h for which the approximation is within .4 of $v(100)$ in absolute value. Compare the number of function evaluations (evaluations of $f(x,v)$) required to compute V_h and the number of function evaluations required for the Euler method. Which method gives fewer function evaluations? (The number of function evaluations is the "cost" of the method.)

Hand in all m-files and a diary of your matlab session.