EM394H/CSE393H ADVANCED MATHEMATICAL THEORY

OF FINITE ELEMENT METHODS

Spring 2019, # 13825/ # 62955 , TTh 12:30-2:00, GDC 2.402

The class covers fundamentals of convergence analysis for Finite Element (FE) method and linear boundary-value problems with a focus on a different set of problems changing every semester. So, in principle, the class may be taken multiple times.

This semester we will cover the fundamentals of the convergence theory for conforming methods using the energy spaces forming the exact sequence: $H^1, H(\text{curl}), H(\text{div})$ and L^2 spaces. *The class is recommended for the first time students.*

We will start by reviewing fundamentals on variational formulations and energy spaces reviewing classical model problems: diffusion-convection-reaction, linear acoustics, Maxwell equations, elastodynamics, Stokes equations, simple coupled problems. Most of the material will be presented w/o proofs but with illustrative examples.

We will continue then with the classical results on abstract variational problems and corresponding fundamental stability and convergence results for the Galerkin method: Ritz theory, Mikhlin theory of asymptotic stability, Cea's Lemma, Babuška's Theorem, Brezzi's theory for mixed problems, and finish with the concept of stabilised methods.

We shall focus then on the exact sequence structure and known polynomial exact sequences corresponding to classical continuous, Nédélec and Raviart-Thomas finite elements, followed by the development of classical abstract interpolation theory and the h- convergence analysis for conforming methods.

The end of the semester will be devoted to more advanced topics: a-priori error estimation for h-adaptive methods, discussion on p- and hp-versions of finite elements.

I will use my 1D and 2D hp codes to illustrate the theoretical estimates.

CLASS OUTLINE SCHEDULE:

Jan 22 - Jan 25 Examples of boundary-value and initial boundary-value problems and variational formulations.

Jan 28 - Feb 1 Well-posedness of a variational problem, Babuška - Nečas Theorem.

Feb 4 - Feb 8 Standard energy spaces: $H^1, H(\text{curl}), H(\text{div}, L^2$ and the exact sequence.

Feb 11 - Feb 15 Abstract variational problem. Galerkin and Ritz methods.

Feb 18 - Feb 22 Coercive problems: Lax-Milgram Theorem and Cea's Lemma.

Feb 25 - Mar 1 Babuška's Theorem and notion of discrete stability.

Mar 4 - Mar 8 Mikhlin's theory and concept of asymptotic stability.

Mar 11 - Mar 15 Brezzi's theory and mixed problems.

Mar 18 - Mar 22 Spring break.

Mar 25 - Mar 29 Construction of H^1-, H(curl)-, H(div-, and L^2-conforming elements.

Apr 1 - Apr 5 Abstract interpolation theory in Sobolev spaces. Bramble-Hilbert Lemma.

Apr 8 - Apr 12 Convergence in weaker norms. Aubin-Nitsche trick.

Apr 15 - Apr 19 Accounting for integration error. First Strang's lemma.

Apr 22 - Apr 26 Convergence analysis for h-adaptive methods.

Apr 29 - May 3 Conforming p and hp finite elements. Projection-based interpolation theory.

May 6 - May 10 Review.

The class will be conducted in a seminar style. No exams (including final) will be given. Instead, problems with varying difficulty, ranging from "hard theory", through practice exercises, to assignments involving numerical experiments , will be assigned in the class on a continuous basis. Each problem will be worth a number of points (5-20). The final grade will be determined by the number of collected points.

Final score range	grade
100 and above	A+
80 - 100	A
60 - 80	B
40 - 60	C
20 - 40	D
00 - 20	F

Discussion session; one and a half hour, once a week, to be determined (most likely Fri, 3:00 - 4:30).

Instructor: Dr. Leszek Demkowicz, POB 6.326, Office hours: immediately after the class.