## Homework 4, Math222a:

Problem 1: Let $A$ be an $m \times n$ matrix of rank $k$. Provided that $m, n, k$ are as in the top row, please answer questions 1 through 6 in the table below.

|  | $m>n>k$ | $m>n=k$ | $m=n=k$ | $m=n>k$ | $n>m=k$ | $n>m>k$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q1: |  |  |  |  |  |  |
| Q2: |  |  |  |  |  |  |
| Q3: |  |  |  |  |  |  |
| Q4: |  |  |  |  |  |  |
| Q5: |  |  |  |  |  |  |
| Q6: |  |  |  |  |  |  |

Q1: Is it the case that $\operatorname{span}(A)=\mathbb{R}^{m}$ ?
Q2: Is it the case that $A \boldsymbol{x}=\boldsymbol{b}$ always has a solution?
Q3: Is the map $\boldsymbol{x} \mapsto A \boldsymbol{x}$ onto?
Q4: Are the columns of $A$ linearly independent?
Q5: Provided that $A \boldsymbol{x}=\boldsymbol{b}$ is consistent, is the solution unique?
Q6: Is the map $\boldsymbol{x} \mapsto A \boldsymbol{x}$ one-to-one?
Problem 2: Consider the map

$$
T\left(\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]\right)=\left[\begin{array}{c}
-x_{1} \\
x_{2}
\end{array}\right] .
$$

- Is $T$ linear?
- If $T$ is linear, what is its standard matrix?
- Is $T$ onto?
- Is $T$ one-to-one?
- Make two sketches similar to Fig. 6 in ch. 1.8 showing the geometric action of $T$.


## Also do:

Section 1.6: 6,8
Section 1.8: 6,20,26,32
Section 1.9: 6,8,10,24,30,36

