Homework set 1 — APPM5450

From the textbook: 7.1, 7.3, 7.4. Optional: 7.2, 7.5.

Problem 1: Suppose that H is a Hilbert space, and that $(\psi_n)_{n=1}^\infty$ is an ON-set in H. Let $\mathcal{P}$ denote the set of finite linear combinations of elements in $(\psi_n)_{n=1}^\infty$. Prove that:

$$(\psi_n)_{n=1}^\infty \text{ is a basis for } H \quad \Leftrightarrow \quad \mathcal{P} \text{ is dense in } H.$$

Problem 2: Suppose that $f, g \in C(\mathbb{T})$. Prove that:

(a) $f * g \in C(\mathbb{T})$.

(b) $f * g = g * f$.