**Notes on Midterm 3**

Midterm 3 covers sections 11.5 – 11.8, and 12.1 – 12.6.

*Sections 11.5 – 11.8:* Everything is included, except the last two paragraphs of Sec. 11.7 (the material on Laplace transforms, and ultra-distributions).

*Section 12.1:* For most of the concepts introduced in this section, it is enough for you to simply be aware that they exist. You should know the definition of a $\sigma$-algebra (Def. 12.1), and the definition of a measure (Def. 12.5), though. You should also know the basic properties of Lebesgue measure (that it is translation invariant, etc), and be familiar with the concepts of "nullsets" and "almost everywhere". In particular, there will be no questions on: Completion of measures, $\sigma$-algebras generated by a collection of sets, probability spaces, Stieltjes measures.

*Section 12.2:* You should be aware of the general definition of a measurable function, and of the fact that pointwise convergence preserves measurability. No exam questions will be based on material from this section, though.

*Section 12.3:* Everything is included except Example 12.31.

*Section 12.4:* Everything is included except Corollary 12.36. Theorem 12.35 is very important, make sure you know how to apply it.

*Section 12.5:* From this section, you only need to know Fubini's theorem when applied to Lebesgue measure on $\mathbb{R}^d$. In particular, you should know that if $\int_{\mathbb{R}} \int_{\mathbb{R}} |f(x,y)|\,dx\,dy < \infty$, then you can evaluate the integral $\int_{\mathbb{R}^2} f(x,y)\,d\mu(x,y)$ via iterated integration in any order. If $\int |f| = \infty$, then you may **not** swap integration order (unless you can justify it by some other means). In particular, you do not need to worry about the question of when the function $f_x : y \mapsto f(x,y)$ is measurable. (To be precise, in real life you may – on very rare occasions – need to worry about such questions, but you will not be tested on it in this class.)

*Section 12.6:* You should know the statements of the results in this section, and you should understand the general idea of the proof of say Theorem 12.50. However, you do not need to memorize the details of the proofs (so that you do not need to learn Urysohn's lemma, or know how to approximate a general Lebesgue measurable set by sets drawn from a countable subset). In other words, there may be exam questions that require you to know that for finite $p$, $L^p(\mathbb{R}^d)$ is separable, and has $C_c^\infty$ as a dense subset. There will not be questions that ask you to prove these statements, though.