APPM 5440, Applied Analysis I, Fall 2012

Gunnar Martinsson
phone: 492-2646, email: martinss@colorado.edu, office: ECOT 233

Meeting times: MWF 11.00 - 11.50, DUAN G131.

Text: The main text is *Applied Analysis* by John K. Hunter and Bruno Nachtergaele. I would also recommend *Introductory Real Analysis* by Kolmogorov and Fomin; it is a beautifully written, very readable book (and cheap, about $11 on Amazon). Finally, if you find that you'd like to refresh your knowledge of advanced calculus, I would recommend *Principles of Mathematical Analysis* by Walter Rudin.

Office hours: Monday 09:00-10:00 and 15:00-16:00. Wednesday 09:00-10:00. These times are *tentative*, and may change once schedules get settled. Please check the course webpage for up-to-date information.

Exams and grading: There will be three midterms and a final. The final is worth 40% of the grade, and the midterms are worth 20% each.

Home work: A weekly home work will be assigned. It will not be collected or graded, but the midterms will consist mostly of problems very similar to the home work problems.

Web-resources: There is a class website with up-to-date information about the syllabus, homework assignments, class handouts, and so on. See:
`http://amath.colorado.edu/faculty/martinss/Teaching/APPM5440_2012f/`

Remark 1: You have probably heard this said many times before, but please keep in mind that for most of us, learning mathematics is a matter of *doing* mathematics rather than reading. So try to work as many of the examples as you can stomach (more than just the assigned home work problems if at all possible). I still often try to learn new material by sitting down comfortably and read, but in my experience, this really only works if the new material is very close to something I already know. Everybody is different, but few of us can learn mathematics without working out examples.

Remark 2: It is very important that you not fall behind in this class. Many students find the material challenging, and almost all concepts introduced will build on concepts that were presented earlier in the semester. If you find that you do not understand, please ask for help as early as possible!

Scope: In this two-semester class we will cover a collection of topics from real analysis, functional analysis, measure and integration theory, and Fourier analysis. The choice of topics is guided by the needs of an applied mathematician or a mathematically inclined engineer or scientist. However, the principal object of the class is to teach mathematics; the goal is to learn not only the results that are covered, but also to learn how to construct a correct mathematical proof. We will from time to time mention application areas, and connect the abstract results to real world phenomena, but this is by no means the main focus of the course.

The techniques we will learn will be applicable to a wide range of topics (probability theory, signal processing, economics, quantum physics, etc) but in order to give a very brief idea of the essence of the class, let us assume that we wish to determine whether a given partial differential equation (such as the Maxwell equations, the Shrödinger equation, the Navier-Stokes equation, ...) has a solution, and if so, determine what properties such a solution will have (is it a smooth function? a bounded function? ...). A very successful technique for doing so has proved to be to introduce sets of functions within which we believe that a solution should be located (such as: "the set of all continuously differentiable functions on an interval", "the set of all probability distributions for the locations of the two electrons in a Helium atom", "the set of all elastic displacements of a plate that is clamped along one edge", ...) and to study the geometry of such sets. The difficulty is that such sets are almost invariably infinite dimensional, and it turns out that concepts such as "distance between points", "open and closed sets", "convergence of sequences", "compactness", etc, which have fairly obvious meanings in $\mathbb{R}^n$, are not trivial to define in more general situations (there are, as we shall see, several different ways of doing so). In some sense, the task of defining and describing the geometry of infinite dimensional spaces, can be said to form the core of the class.

Specifically, we will during the Fall semester cover the following topics: metric and normed spaces, continuous functions, the contraction mapping theorem, the implicit function theorem, topological spaces, Banach spaces, Hilbert spaces.

During the Spring semester, we will cover: Fourier series, bounded linear operators on Hilbert space, spectral theory for bounded linear operators (with a focus on compact and/or self-adjoint ones), the Fourier transform, distributions, measure theory and L^p spaces, Sobolev spaces.

Disability: If you qualify for accommodations because of a disability, please submit to your professor a letter from Disability Services in a timely manner (for exam accommodations provide your letter at least one week prior to the exam) so that your needs can be addressed. Disability Services determines accommodations based on documented disabilities. Contact Disability Services at 303-492-8671 or by e-mail at dsinfo@colorado.edu.

If you have a temporary medical condition or injury, see Temporary Medical Conditions: Injuries, Surgeries, and Illnesses guidelines under Quick Links at Disability Services website and discuss your needs with your professor.

Religious observances: Campus policy regarding religious observances requires that faculty make every effort to deal reasonably and fairly with all students who, because of religious obligations, have conflicts with scheduled exams, assignments or required attendance. In this class, please see the instructor in case of a conflict at least 10 days in advance (preferably earlier). See full details at `http://www.colorado.edu/policies/fac_relig.html`

Classroom behavior: Students and faculty each have responsibility for maintaining an appropriate learning environment. Those who fail to adhere to such behavioral standards may be subject to discipline. Professional courtesy and sensitivity are especially important with respect to individuals and topics dealing with differences of race, color, culture, religion, creed, politics, veteran's status, sexual orientation, gender, gender identity and gender expression, age, disability, and nationalities. Class rosters are provided to the instructor with the student's legal name. I will gladly honor your request to address you by an alternate name or gender pronoun. Please advise me of this preference early in the semester so that I may make appropriate changes to my records. See policies at `http://www.colorado.edu/policies/classbehavior.html` and at `http://www.colorado.edu/studentaffairs/judicialaffairs/code.html#student_code`

Sexual harassment: The University of Colorado Boulder (CU-Boulder) is committed to maintaining a positive learning, working, and living environment. The University of Colorado does not discriminate on the basis of race, color, national origin, sex, age, disability, creed, religion, sexual orientation, or veteran status in admission and access to, and treatment and employment in, its educational programs and activities. (Regent Law, Article 10, amended 11/8/2001). CU-Boulder will not tolerate acts of discrimination or harassment based upon Protected Classes or related retaliation against or by any employee or student. For purposes of this CU-Boulder policy, "Protected Classes" refers to race, color, national origin, sex, pregnancy, age, disability, creed, religion, sexual orientation, gender identity, gender expression, or veteran status. Individuals who believe they have been discriminated against should contact the Office of Discrimination and Harassment (ODH) at 303-492-2127 or the Office of Student Conduct (OSC) at 303-492-5550. Information about the ODH, the above referenced policies, and the campus resources available to assist individuals regarding discrimination or harassment can be obtained at `http://www.colorado.edu/odh`.

Honor code: All students of the University of Colorado at Boulder are responsible for knowing and adhering to the academic integrity policy of this institution. Violations of this policy may include: cheating, plagiarism, aid of academic dishonesty, fabrication, lying, bribery, and threatening behavior. All incidents of academic misconduct shall be reported to the Honor Code Council (honor@colorado.edu; 303-735-2273). Students who are found to be in violation of the academic integrity policy will be subject to both academic sanctions from the faculty member and non-academic sanctions (including but not limited to university probation, suspension, or expulsion). Other information on the Honor Code can be found at `http://www.colorado.edu/policies/honor.html` and at `http://www.colorado.edu/academics/honorcode/`