

Homework set 8 — APPM5440

From the textbook: 3.6, 3.7.

On the next page, you'll find last year's midterm. Problem 4 on that midterm is part of this week's homework.

The remaining homework problems draw on Monday's lecture, and are not part of the material included in the midterm.

Problem 1: Let X be a set with infinitely many members. We define a collection \mathcal{T} of subsets of X by saying that a set $\Omega \in \mathcal{T}$ if either $\Omega^c = X \setminus \Omega$ is finite, or if Ω is the empty set. Verify that \mathcal{T} is a topology on X . This topology is called the “co-finite” topology on X . Describe the closed sets.

Problem 2: Let X denote a finite set, and let \mathcal{T} be a metrizable topology on X . Prove that \mathcal{T} is the discrete topology on X .

Problem 3: Consider the set $X = \{a, b, c\}$, and the collection of subsets $\mathcal{T} = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Is \mathcal{T} a topology? Is \mathcal{T} a metrizable topology?

Applied Analysis (APPM 5440): Midterm 2

5.00pm – 6.30pm, Oct 26, 2005. Closed books.

Problem 1: State the Arzelà-Ascoli theorem.

Problem 2: We describe two mathematical objects below. For each description, either provide an example of such an object, or explain why it does not exist. (Brief answers, please!)

- (1) A collection of functions in $C(\mathbb{R})$ that is equicontinuous but not uniformly equicontinuous.
- (2) A collection of functions in $C([0, 1])$ that is equicontinuous but not uniformly equicontinuous.

Problem 3: Let $X = \{a, b, c, d\}$ be a set, and let

$$\mathcal{S} = \{\emptyset, X, \{b, d\}, \{a, c\}, \{d\}, \{a, b, c\}\}$$

be a collection of subsets of X .

- (1) Prove that \mathcal{S} is not a topology on X .
- (2) Let \mathcal{T} denote the smallest topology on X that contains \mathcal{S} (in other words, \mathcal{T} is the topology generated by the sub-base \mathcal{S}). List the sets that are contained in \mathcal{T} but not in \mathcal{S} .

Problem 4: Consider the integral equation

$$(*) \quad u(x) = \pi^2 \sin(x) + \frac{3}{2} \int_0^{\cos(x)} |x - y| u(y) dy.$$

Prove that (*) has a unique solution in $C([0, 1])$.

Problem 5: Let X be a topological space, let Y be a Hausdorff space, and let f and g be continuous maps from X to Y . Is it necessarily the case that the set $\Omega = \{x \in X : f(x) = g(x)\}$ is closed? Justify your answer by either giving a proof or a counter example.