APPM 5440, Applied Analysis I, Fall 2005

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Meeting times: MWF 12.00 - 12.50, Duane Physics Building, Room G2B21. (See also the section on "tutorial sessions" below.)

Text: The main text is "Applied Analysis" by John K. Hunter and Bruno Nachtergaele. I would also recommend "Introductory Real Analysis" by Kolmogorov and Fomin; it is a beautifully written, very readable book (and cheap, about $11 on Amazon). Finally, if you find that you'd like to refresh your knowledge of advanced calculus, I would recommend "Principles of Mathematical Analysis" by Walter Rudin.

Office hours: Mondays 13.00 – 14.00, Wednesdays 16.00 – 17.00. If you cannot make the office hours, feel free to make an appointment for another time.

Exams and grading: There will be three midterms and a final. The final is worth 55% of the grade, and the midterms are worth 15% each.

Home work: A weekly home work will be assigned. It will not be collected or graded, but the midterms will consist mostly of problems very similar to the home work problems.

Tutorial sessions: Wednesdays 17.00 – 18.00. The main purpose of this session is to discuss the home work problems for the previous week.

Web-resources: There is a class website with up-to-date information about the syllabus, homework assignments, class handouts, and so on. It will soon be located at:
`http://amath.colorado.edu/courses/5440/2005fall/`
As a temporary solution until the correct URL is working, you'll find a copy at:
`http://www.math.yale.edu/users/pjm34/APPM5440`

Remark 1: You have probably heard this said many times before, but please keep in mind that for most of us, learning mathematics is a matter of *doing* mathematics rather than reading. So try to work as many of the examples as you can stomach (more than just the assigned home work problems if at all possible). I still fall for the temptation myself of sitting down in a comfortable couch and try to learn new material by simply reading, but it really only works if the new material is very close to something I already know. Everybody is different, but almost all mathematicians I know function in the same way (with the caveat that what would be "very close" for someone like Kolmogorov, would not seem "very close" to me).

Remark 2: It is very important that you not fall behind in this class. Almost all material that will be covered will utilize results that were presented earlier in the semester. If you find that you do not understand, please ask for help as early as possible!

Scope: In this two-semester class we will cover a collection of topics from real analysis, functional analysis, measure and integration theory, and Fourier analysis. The choice of topics is guided by the needs of an applied mathematician (or, equivalently, the needs of a mathematically inclined engineer or scientist). However, the principal object of the class is to teach mathematics; the goal is to learn not only the results that are covered, but also to learn how to construct a correct mathematical proof. We will from time to time mention application areas, and connect the abstract results to real world phenomena, but this is by no means the main focus of the course.

The techniques we will learn will be applicable to a wide range of topics (probability theory, signal processing, economics, quantum physics, etc) but in order to give a very brief idea of the essence of the class, let us assume that we wish to determine whether a given partial differential equation (such as the Maxwell equations, the Shrödinger equation, the Navier-Stokes equation, ...) has a solution, and if so, determine what properties such a solution will have (is it a smooth function? a bounded function? ...). The most successful technique for doing so has proved to be to introduce sets of functions within which we believe that a solution should be located (such as: "the set of all continuously differentiable functions on an interval", "the set of all probability distributions for the locations of the two electrons in a Helium atom", "the set of all elastic displacements of a plate that is clamped along one edge", ...) and to study the geometry of such sets. The difficulty is that such sets are almost invariably infinite dimensional, and it turns out that concepts such as "distance between points", "open and closed sets", "convergence of sequences", "compactness", etc, which have fairly obvious meanings in $\mathbb{R}^n$, are not trivial to define in more general situations (there are, as we shall see, several different ways of doing so). In some sense, the task of defining and describing the geometry of infinite dimensional spaces, can be said to form the core of the class.

Specifically, we will during the Fall semester cover the following topics: metric and normed spaces, continuous functions, the contraction mapping theorem, the implicit function theorem, topological spaces, Banach spaces, Hilbert spaces, Fourier series.

During the Spring semester, we will cover: bounded linear operators on Hilbert space, spectral theory for bounded linear operators (compact and/or self-adjoint ones only), the Fourier transform, distributions, measure theory and L^p spaces, Sobolev spaces.