Quiz:

Feel free to hand this in anonymously.

Question 1: Mark in each slot:

- "no" if the value does not exist
- $\pm\infty$ if the value is infinite
- "finite" if the value exists and is finite (give an exact value if you know it but don't spend time on trying to figure it out).

	$\lim_{n\to\infty} x_n$	$\sup\{x_n\}_{n=1}^{\infty}$	$\limsup_{n\to\infty} x_n$
$x_n = 1/n$			
$x_n = (-1)^n + \sin(n)/n$		skip	
$x_n = \sum_{j=1}^{n} 1/j$			
$x_n = \sum_{j=1}^{n} 1/j - \log(n)$		skip	skip
$x_n = \sum_{j=1}^{n} 1/j^2$			
$x_n = \sum_{j=1}^{n} (-1)^j/j$			
$x_n = \sum_{j=1}^{n} (-1)^j/j^2$			

Question 2: Circle the sums that are absolutely convergent:

$$\sum_{j=1}^{n} 1/j, \qquad \sum_{j=1}^{n} 1/j^2, \qquad \sum_{j=1}^{n} (-1)^j/j, \qquad \sum_{j=1}^{n} (-1)^j/j^2.$$

Question 3: Let α denote a real number, let $B = \{x \in \mathbb{R}^2 : \ |x| \leq 1\}$ and set

$$f(\alpha) = \int_B \frac{1}{|x|^\alpha}\, dA.$$

(a) For which values of α is $f(\alpha)$ finite?

(b) What is the answer if B is the unit ball in $\mathbb{R}^n$ rather than $\mathbb{R}^2$?

Question 4: Let f be a continuous function defined on the set Ω. For each of the examples of sets given below, answer the following questions: Is f necessarily bounded? Is f necessarily uniformly continuous? (Give a counter examples if the answer is no.)

(a) $\Omega = \{x \in \mathbb{R}^2 : \ |x| \leq 2\}$.

(b) $\Omega = \{x \in \mathbb{R}^2 : \ 0 < |x| \leq 2\}$.

(c) $\Omega = \{x \in \mathbb{R}^2 : \ |x| \geq 2\}$.

(d) $\Omega = \cup_{n=1}^{\infty} [1/n,\, 1/n + 1/n^3]$.

Question 5: Let $\{F_n\}_{n=1}^{\infty}$ be a sequence of closed sets in $\mathbb{R}^2$ and let $\{G_n\}_{n=1}^{\infty}$ be a sequence of open sets in $\mathbb{R}^2$. Which of the following four sets are necessarily open? Necessarily closed?

(a) $\cup_{n=1}^{\infty} F_n$

(b) $\cap_{n=1}^{\infty} F_n$

(c) $\cup_{n=1}^{\infty} G_n$

(d) $\cap_{n=1}^{\infty} G_n$

Question 6: The parallelogram law in $\mathbb{R}^n$ says that for any $x, y \in \mathbb{R}^n$

$$|x+y|^2 + |x-y|^2 =$$

Question 7: Let Ω be a bounded set in $\mathbb{Q}$ (the set of rational numbers). Does the set Ω necessarily have a least upper bound in $\mathbb{Q}$? If no, give a counter example.

Question 8: Let Ω be a closed set in $\mathbb{R}^3$ (not necessarily bounded) and let $\{x_n\}_{n=1}^{\infty}$ denote a Cauchy sequence in Ω. Does x_n necessarily have a limit value in Ω? If no, give a counter example.

Question 9: Let A be an $n \times n$ matrix of real numbers. Give a sufficient condition for there to exist a unitary matrix U, and a diagonal matrix D such that $A = U\,D\,U^{\mathrm{T}}$.

Question 10: Let f be a continuous function on the interval $[-\pi,\,\pi]$ and define for $n = \dots, -2, -1, 0, 1, 2, \dots$ the complex number a_n by

$$a_n = \int_{-\pi}^{\pi} e^{inx} f(x)\,dx.$$

Give the right hand side of the following equality:

$$\sum_{n=-\infty}^{\infty} |a_n|^2 =$$