Exam #1 **APPM 2360** **Spring 2008**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answer. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. A one-page crib sheet is allowed.

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1. (*15 points, 5 each*) Consider the following equation

$$\frac{dy}{dt} = ty^2$$

(a) Find the general solution of the equation. Are there any solutions that are not included in this solution? Explain.

(b) Find the time $t_A > 0$ for which the solution with $y(t_0) = y_0$ has a vertical asymptote. **Hint**: A vertical asymptote is the time for which $y(t_A) \to \infty$.

(c) Find the only solution of the equation that does *not* have a vertical asymptote.

2. (*28 points, 7 each*) Consider the differential equation

$$\frac{dy}{dt} = 3t^2 y + 3t^5$$

(a) Show that $y_p = -t^3 - 1$ is a particular solution of the equation.

(b) Find the general solution of the equation.

(c) Define the new variable $x = t^n$ and find the integer n such that the differential equation becomes

$$\frac{dy}{dx} = y + x$$

(d) Solve the equation for $y(x)$ and show that is consistent with your solution in part (b).

3. (*21 points, 7 each*) Consider the differential equation

$$L[y] = \frac{dy}{dt} + y \ln y - e^t y = 0$$

(a) Make the substitution $u = \ln y$ and show that $u = u(t)$ satisfies an equation of the form $M[u] = f(t)$. Define the operator M and the function $f(t)$.

(b) Formally prove that L is *not* a linear operator, while M is a linear operator.

(c) Use your results in part (a) to construct the general solution of $L[y] = 0$.

4. (*21 points, 7 each*) A body moves through a resisting medium with resistance proportional to its velocity $v = v(t)$ so that

$$\frac{dv}{dt} = -kv, \quad v(0) = v_0$$

where $k > 0$.

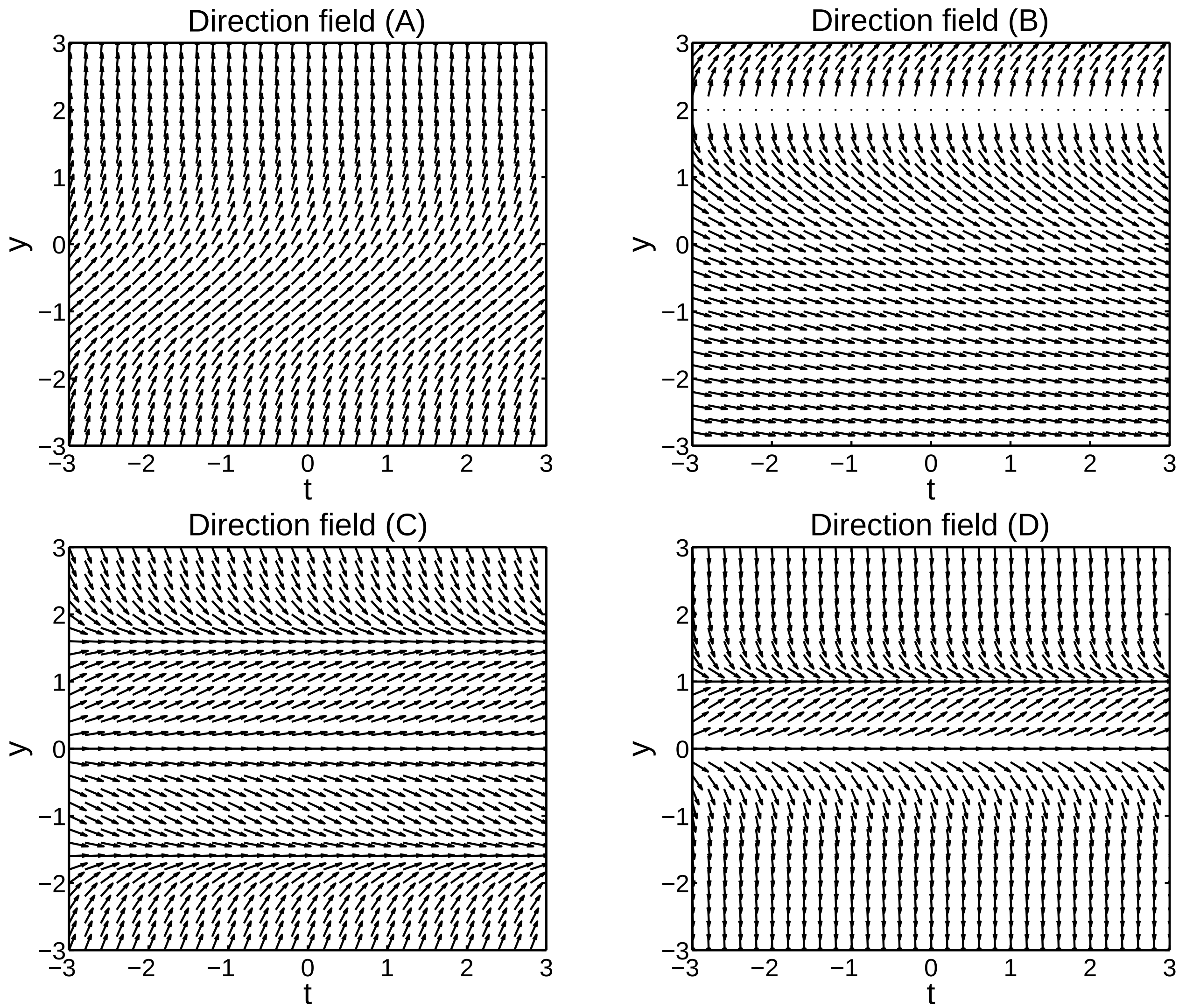
(a) Find the velocity $v(t)$ of the body for all t.

(b) Find the position $x = x(t)$ of the body for all t, if $x(0) = x_0$. Recall $v = dx/dt$.

(c) Find the position of the body when it stops.

More on the back; turn over the page.

5. (*15 points, 5 each*) Consider the following direction fields that correspond to four different first order ordinary differential equations:



(a) Match the following differential equations to the direction fields:

(i) $\dfrac{dy}{dt} = 3y(1-y)$ (ii) $\dfrac{dy}{dt} = y\ \cos y$ (iii) $\dfrac{dy}{dt} = \dfrac{1}{y-2}$ (iv) $\dfrac{dy}{dt} = y^2 + 2y + 2$

(b) Classify the equations in part (a) according to linear or nonlinear.

(c) Identify the equilibria shown in the graphs and classify them according to stable or unstable.

Good Luck!!!