APPM2360 — Differential Equations with Linear Algebra

Instructor: Gunnar Martinsson

Course syllabus: Pick one up today!

Instructor availability: Office hours MW 11-12, W 3-4 in ECOT 233.
You can go to any instructor's or TA's office hours. (Listed on webpage.)
Email me at `martinss@colorado.edu` — please write "APPM2360" in subject!

Course webpage: `http://amath.colorado.edu/courses/2360/2009Spr/`

- Course syllabus.
- Homework schedule.
- Old exams and other resources.
- Announcements.

Grading policy:

Homeworks:	150 points
Projects:	150 points (3×50 points)
Section exams:	300 points (3×100 points)
Final:	200 points
Total:	800 points

Engineering majors must achieve a grade of C- or better (C for Aerospace). If you obtain 70% of the points you will have a grade no worse than C- (if we grade on a curve, it will be to your advantage). However, in order **to attain a grade of C- or better in the course, you must achieve a grade of at least C- (i.e., 70%) on the exams (overall)**, regardless of your homework and project grades.

You can check your grades on `http://culearn.colorado.edu/`

Requests for regrades must be made in writing within 2 weeks.

Exams are closed books — but a double sided crib sheet is allowed.

Exhortation for the improvement of morals:

It is extremely important that you do exercises.

Some problems are marked as homework problems and will be collected.
You should do many more problems than just those!

Every chapter starts with a number of "basic" exercises — if you understand the material of the chapter, these should be easy. Note that most of the exam problems will be harder than these questions.

Almost nobody can learn math by just reading text and attending lectures (I cannot, for sure). Again: It is extremely important that you do exercises!

Review — derivatives (exhortation for improved morals continued)

In order to understand this class it is very important that you understand the concept of a derivative well — review calculus NOW if you need to!

Formal definition:

Let y be a real valued function of a real variable. The *derivative* of y at t is

$$y'(t) = \lim_{h \to 0} \frac{y(t) - y(t+h)}{h},$$

if the limit exists. (If it does not, then y is not differentiable at t and the derivative is not defined.)

Other notation: $y'(t) = \dot{y}(t) = \dfrac{dy}{dt} = y_t$.

Informally speaking, the derivative marks the slope of the graph of y at t.

Let us consider an equation that involves the derivative:

$$y'(t) = t^3.$$

We easily solve this equation via integration. Simply fix some number t_0, then

$$\int_{t_0}^{t} y'(s)\, ds = \int_{t_0}^{t} s^3\, ds.$$

Evaluating both sides we obtain

$$y(t) - y(t_0) = \frac{1}{4}t^4 - \frac{1}{4}t_0^4.$$

We can also write

$$y(t) = \frac{1}{4}t^4 + C,$$

where

$$C = y(t_0) - \frac{1}{4}t_0^4.$$

Easy calculus exercise!

So we could easily solve

$$y'(t) = t^3. \tag{1}$$

Now consider the equation

$$y'(t) = 2\,y(t). \tag{2}$$

Can you solve (2)?

What makes (2) much harder is that the right hand side involves y as well. This makes it a *differential equation*. In this course, we will learn different methods for solving equations such as (2).

- Analytic — use math skills to find a solution given by a formula.
  Great when it works — but it only works for simple equations.

- Geometrical — gives you a sense of what the solution looks like.
  It gives you "qualitative" information (as opposed to "quantitative").

- Numerical — construct an approximate solution using a computer.
  This is how you typically solve ODEs in "real life."

By the way, the solution of

$$y'(t) = 2\,y(t),$$

is

$$y(t) = C\,e^{2\,t},$$

for any real number C.

Definition:

In the equation above, y is the dependent variable,
and t is the independent variable.

Examples:

$$y'(t) - 2\,y(t) = 0$$

$$y'(t) - t^2\,y(t) = 0$$

$$y'(t) + (y(t))^2 = 0$$

$$y''(t) + y'(t) + y(t) = 0$$

Definition: The "order" of the equation is the degree of the highest derivative.

Examples:

$$y'(t) - 2\,y(t) = 0 \quad \text{first order}$$

$$y'(t) - t^2\,y(t) = 0 \quad \text{first order}$$

$$y'(t) + (y(t))^2 = 0 \quad \text{first order}$$

$$y''(t) + y'(t) + y(t) = 0 \quad \text{second order}$$

Definition: The "order" of the equation is the degree of the highest derivative.

Some differential equations involve functions of many variables; say $y = y(x, t)$.

$$\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{Laplace's equation}$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{\partial y}{\partial t} = 0 \qquad \text{Heat equation}$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{Wave equation}$$

$$\frac{\partial^2 y}{\partial x^2} + k^2\, y = 0 \qquad \text{Helmholtz' equation}$$

$$\frac{\partial^2 y}{\partial x^2} - y\,\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t} = 0 \qquad \text{Burger's equation}$$

$$-\frac{\partial^2 y}{\partial x^2} + V\, y - i\,\frac{\partial y}{\partial t} = 0 \qquad \text{Schrödinger equation}$$

An ordinary differential equation involves derivatives with respect to one variable.
A partial differential equation involves derivatives with respect to many variables.

Elasticity $$\frac{1}{2}E_{ijkl}\left(\frac{\partial^2 u_k}{\partial x_l \partial x_j} + \frac{\partial^2 u_l}{\partial x_k \partial x_j}\right) = f_i,$$

Stokes $$\Delta \mathbf{u} = \nabla p, \qquad \nabla \cdot \mathbf{u} = 0,$$

Maxwell $$\begin{cases} \nabla \cdot \mathbf{E} = \rho & \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 & \nabla \times \mathbf{B} = \mathbf{J} + \dfrac{\partial \mathbf{E}}{\partial t} \end{cases}$$

Not to worry — this class concerns **ordinary** differential equations only!

Even just ordinary equations exhibit interesting behavior, though.

Consider for instance the following equation:

$$y''(t) + 4\,y(t) = 0, \qquad y(0) = 1, \qquad y'(0) = 0.$$

This equation models a simply mass-spring system.

The solution is

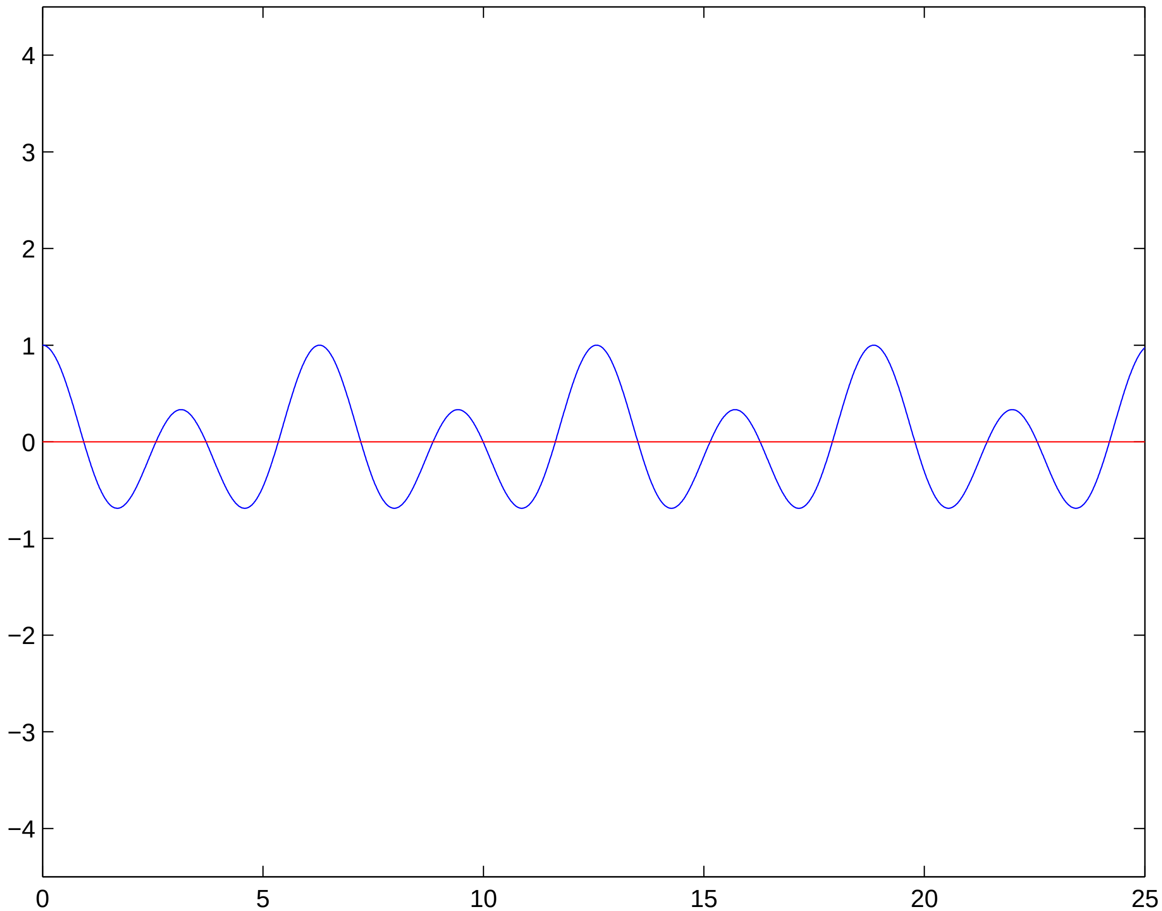
$$y(t) = \cos(2\,t).$$

Now consider a forced system:

$$y''(t) + 4\,y(t) = \cos(t), \qquad y(0) = 1, \qquad y'(0) = 0.$$

The solution is

$$y(t) = \frac{2}{3}\,\cos(2\,t) + \frac{1}{3}\,\cos(t).$$

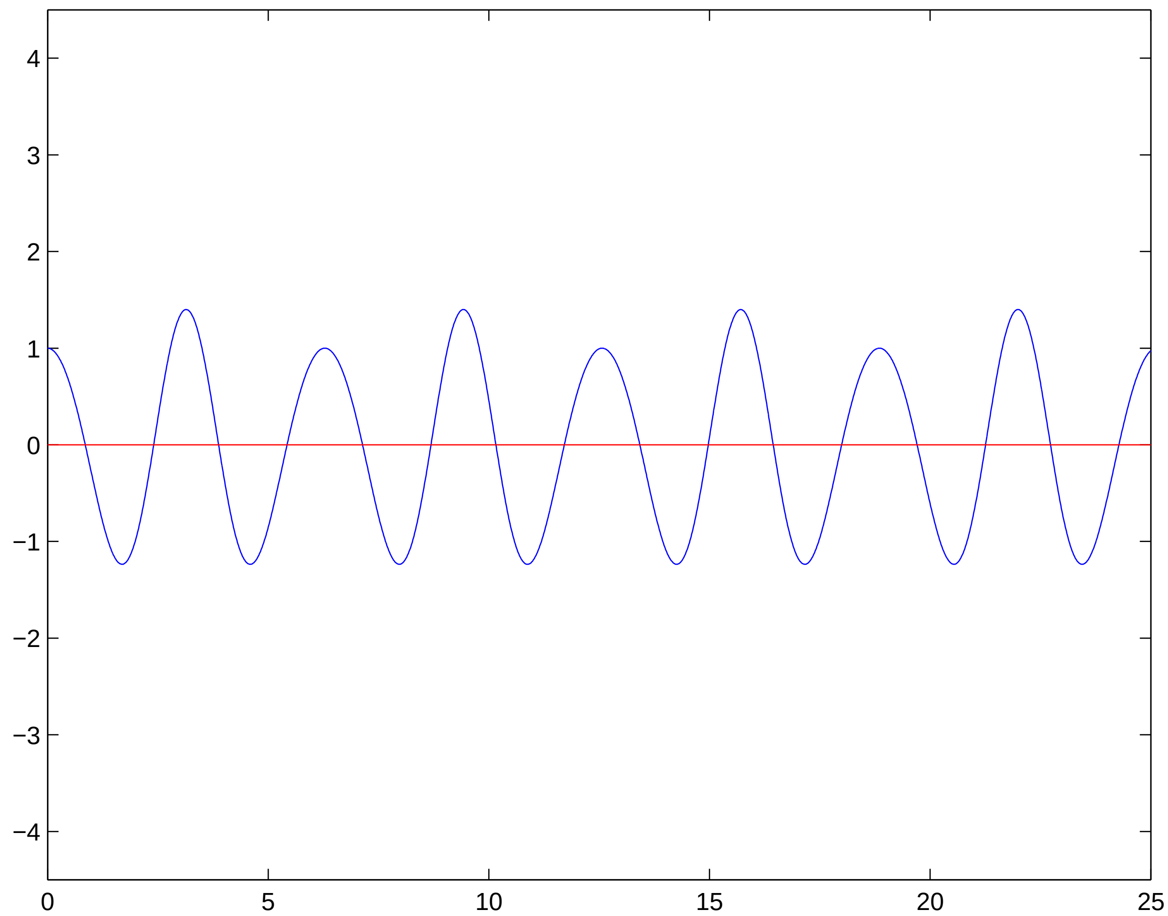


Let us change the frequency

$$y''(t) + 4\,y(t) = \cos(3\,t), \qquad y(0) = 1, \qquad y'(0) = 0.$$

The solution is

$$y(t) = \frac{6}{5}\,\cos(2\,t) - \frac{1}{5}\,\cos(3\,t).$$

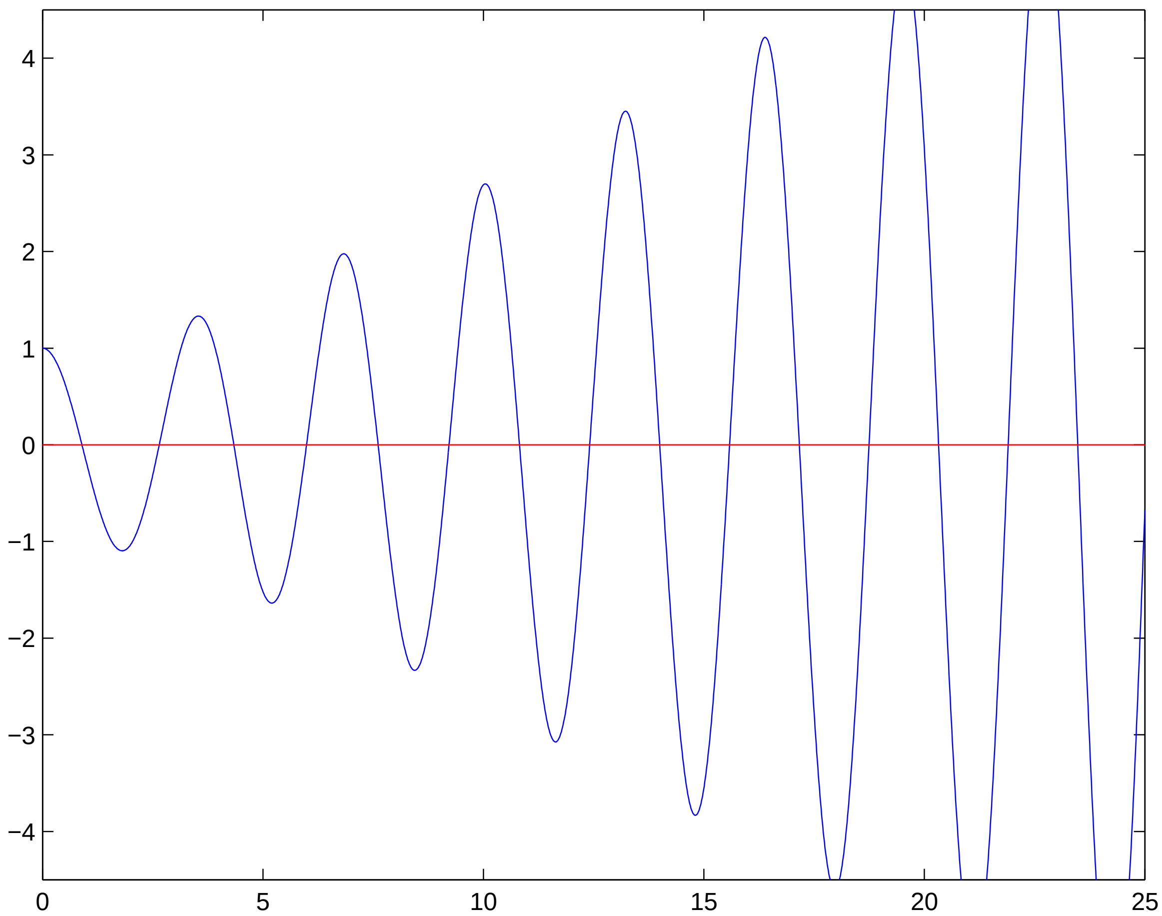


Let us change the frequency again ...

$$y''(t) + 4\,y(t) = \cos(2\,t), \qquad y(0) = 1, \qquad y'(0) = 0.$$

The solution is

$$y(t) = \cos(2\,t) + \frac{1}{4}\,t\,\sin(2\,t).$$



Key points:

- What is a differential equation?
- *Order* of a differential equation.
- Classification ordinary / partial differential equations.
- The terms “dependent variable” and “independent variable.”

A final note:

This lecture will be posted on the course webpage.

This will not be the rule, but may happen from time to time.

Theorem: *If attendance decreases when lecture notes are posted, then the frequency with which lecture notes are posted will decrease.*