Week 9 HW §4.1, 4.2

$\boxed{4.1.8}$ $4\ddot{x} + \pi^2 x = 0$, $x(0)=1$, $\dot{x}(0)=\pi$

$$\omega_0 = \sqrt{\frac{4\pi^2}{4}} = \frac{\pi}{2}$$

$$x(t) = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$$

$$x(0) = c_1 = 1$$

$$\dot{x}(t) = -\omega_0 \sin \omega_0 t + c_2 \omega_0 \cos \omega_0 t$$

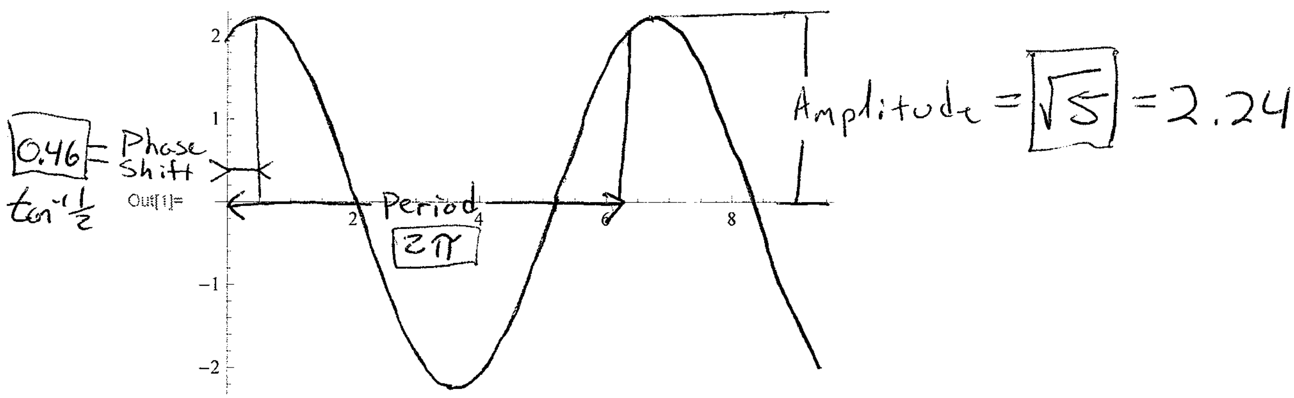
$$\dot{x}(0) = c_2 \omega_0 = c_2 \cdot \frac{\pi}{2} = \pi \Rightarrow c_2 = 2$$

$$x(t) = \cos \omega_0 t + 2 \sin \omega_0 t$$

$$\boxed{x(t) = \cos\left(\frac{\pi}{2}t\right) + 2\sin\left(\frac{\pi}{2}t\right)}$$

4.1.10

In[1]:= Plot[2 Cos[t] + Sin[t], {t, 0, 3 * Pi}]



(c) $x(t) = (\sqrt{5}) \cos(t - 0.46)$

4.1.28

$\ddot{x} + 16x = 0, \quad x(0) = 0, \quad \dot{x}(0) = 4$

$\boxed{\omega_0 = \sqrt{16} = 4}$

$x(t) = c_1 \cos 4t + c_2 \sin 4t$

$x(0) = c_1 = 0$

$\dot{x}(t) = c_2 \cdot 4 \cos 4t$

$\dot{x}(0) = 4c_2 = 4 \implies c_2 = 1$

$x(t) = \sin 4t$

$\boxed{A = 1}$

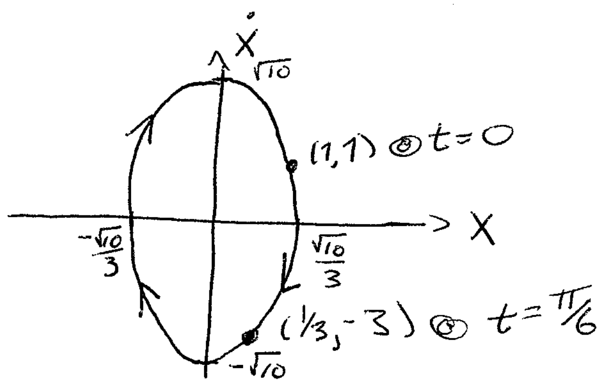
$c_1 = A\cos\delta, \quad c_2 = A \sin\delta$ (From p. 200, (8))

$\implies \boxed{\delta = \pi/2}$

$\boxed{4.1.34}$ $\ddot{x} + 9x = 0$, $x(0) = 0$, $\dot{x}(0) = 0$

$$x(t) = \cos(3t) + \frac{1}{3}\sin(3t)$$

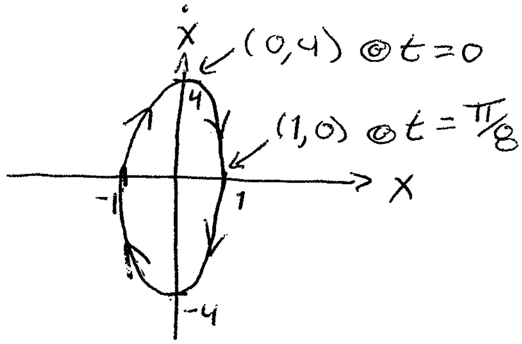
$$\dot{x}(t) = -3\sin(3t) + \cos(3t)$$



4.1.37 $\ddot{x} + 16x = 0,\ x(0) = 0,\ \dot{x}(0) = 4$

$$x(t) = \sin(4t)$$

$$\dot{x}(t) = 4\cos(4t)$$



4.2.9 $2y'' - 3y' + y = 0$

$$r_1, r_2 = \frac{-(-3) \pm \sqrt{(-3)^2 - 4\cdot 2\cdot 1}}{2(2)} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3}{4} \pm \frac{1}{4}$$

let $r_1 = \frac{1}{2},\ r_2 = 1$

then $\boxed{y(t) = c_1 e^{t/2} + c_2 e^{t}}$

$\boxed{4.2.17}$ $y'' + 2y' + y = 0, \quad y(0) = 0, \quad y'(0) = 1$

$$r_1, r_2 = \frac{-2 \pm \sqrt{2^2 - 4(1)(1)}}{2(1)} = -1$$

$\Rightarrow r_1 = r_2 = -1$

$\Rightarrow$ basis is $\{e^{-t}, te^{-t}\}$

$y(t) = c_1 e^{-t} + c_2 t e^{-t}$

$y(0) = c_1 = 0$

$y'(t) = c_2 t(-e^{-t}) + c_2 e^{-t}$

$y'(0) = c_2 = 1$

$$\boxed{y(t) = te^{-t}}$$

$\boxed{4.2.25}$ $5y'' - 10y' - 15y = 0$

divide by 5

$y'' - 2y' - 3y = 0$

Characteristic eqn:

$r^2 - 2r - 3 = 0$

$(r-3)(r+1) = 0$

let $r_1 = 3$

$r_2 = -1$

$$\boxed{\text{Basis is } \{e^{3t}, e^{-t}\}}$$

$$\boxed{\text{Solution Space}: \{y \mid y = c_1 e^{3t} + c_2 e^{-t};\ c_1, c_2 \in \mathbb{R}\}}$$

$\boxed{4.2.28}$ $y'' = 0$

Just integrate to solve

$y' = c_1$

$y(t) = c_1 t + c_2$

The solution space theorem says the solution space $\mathbb{S}$ of a 2^{nd} order, homogeneous ODE has dimension 2. Therefore, if two solutions are linearly independent they form a basis.

① $\{1, t\}$

$$W = \begin{vmatrix} 1 & t \\ 0 & 1 \end{vmatrix} = 1 \neq 0 \implies ① \text{ is a basis}$$

② $\{(t+1), (t-1)\}$

$$W = \begin{vmatrix} t+1 & t-1 \\ 1 & 1 \end{vmatrix} = 2 \neq 0 \implies ② \text{ is a basis}$$

③ $\{2t, (3t-1)\}$

$$W = \begin{vmatrix} 2t & 3t-1 \\ 2 & 3 \end{vmatrix} = 2 \neq 0 \implies ③ \text{ is a basis}$$

$\boxed{4.2.30}$ ① $\{te^{-5t}, e^{5t}, 2e^{5t}-1\}$

1st derivatives: $(-5te^{-5t}+e^{-5t}, 5e^{5t}, 10e^{5t})$

2nd derivatives: $(25te^{-5t}-10e^{-5t}, 25e^{5t}, 50e^{5t})$

$$W = \begin{vmatrix} te^{-5t} & e^{5t} & 2e^{5t}-1 \\ -5te^{-5t}+e^{-5t} & 5e^{5t} & 10e^{5t} \\ 25te^{-5t}-10e^{-5t} & 25e^{5t} & 50e^{5t} \end{vmatrix}$$

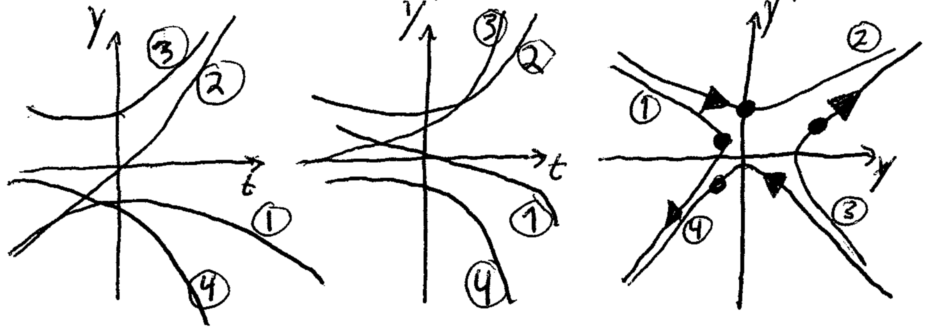
$$W = e^{5t}\left[t\begin{vmatrix} 5 & 10 \\ 25 & 50 \end{vmatrix} - 1\begin{vmatrix} 5t+1 & 10 \\ 25t+10 & 50 \end{vmatrix} + (2-e^{-5t})\begin{vmatrix} 5t+1 & 5 \\ 25t+10 & 25 \end{vmatrix}\right]$$

$= 25e^{5t} \neq 0$

$\Rightarrow$ ① is a basis for the solution space of

$y''' - 10y'' + 25y' = 0$

$\boxed{4.2.55}$



• $\Rightarrow t=0$

▶ $\Rightarrow$ increasing t