HW 6

Sect. 3.1

5.) $BA = \begin{bmatrix} 5 & 3 & 9 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

6.) $CD = \begin{bmatrix} 3 & -1 & 0 \\ 8 & -1 & 2 \\ 9 & 2 & 6 \end{bmatrix}$

8.) $(DC)^T = \begin{bmatrix} 1 & 6 \\ -1 & 7 \end{bmatrix}$

10.) $\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} \Rightarrow$

$$\begin{array}{r} -4(a + 2c = 2) \\ 4a + c = 1 \\ \hline 0 - 7c = -7 \\ c = 1 \\ a + 2 = 2 \Rightarrow a = 0 \end{array} \qquad \begin{array}{r} -4(b + 2d = 0) \\ 4b + d = 4 \\ \hline 0 - 7d = 4 \\ d = -4/7 \\ b = 8/7 \end{array}$$

So $C = \begin{bmatrix} 0 & 8/7 \\ 1 & -4/7 \end{bmatrix}$

[redacted] Consider $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 1 & 4 \end{bmatrix}$

62.) Consider $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $C = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

Then $AB = AC = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ but unless $a = e$, $b = f$, $c = g$, and $d = h$,

$B \neq C$.

Sect. 3.2

1.) $\begin{bmatrix} 1 & 2 & 1 \\ 1 & -3 & 3 \\ 0 & 4 & -5 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ and $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 1 & -3 & 3 & 1 \\ 0 & 4 & -5 & 3 \end{array}\right]$

19.) RREF

[redacted] $\,-3\begin{bmatrix} 1 & 2 & 3 & 1 \\ 3 & 7 & 10 & 4 \\ 2 & 4 & 6 & 2 \end{bmatrix} \to -2\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 2 & 4 & 6 & 2 \end{bmatrix} \to -1\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \to -2\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\to \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Pivot columns: $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 3 & 7 & 10 & 4 \\ 2 & 4 & 6 & 2 \end{bmatrix}$

32.) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 2 & -1 & 1 & 0 \\ 4 & 1 & 5 & 0 \end{array}\right] \to -4\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & -3 & -3 & 0 \\ 4 & 1 & 5 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & -3 & -3 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ So $x_1 + x_3 = 0 \Rightarrow x_1 = -x_3 = x_2$

$x_2 + x_3 = 0$ Choose $x_1 = r$. Then $x_1 = r$, $x_2 = r$, $x_3 = -r$. ∞ many solns.

52.) $A\bar{x} = \bar{0}$ always works for $\bar{x} = \bar{0}$. Alternately, an inconsistent system has a row of the form $[0\ 0 \ldots 0 \mid a]$ where $a \neq 0$.

[redacted] $\,k\left[\begin{array}{cc|c} 1 & k & 0 \\ k & 1 & 2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & k & 0 \\ 0 & 1-k^2 & 2 \end{array}\right] \to -k\left[\begin{array}{cc|c} 1 & k & 0 \\ 0 & 1 & \frac{2}{1-k^2} \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & \frac{-2k}{1-k^2} \\ 0 & 1 & \frac{2}{1-k^2} \end{array}\right]$

So $k \neq \pm 1$. Everything else works.