Homework 14

Sec. 6.4 – 3, 5, 17 and Sec. 7.1 – 14, 18, 23, 32

#3) $\vec{x}' = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \vec{x}$ (star node)

The matrix is triangular so eigenvalues are -2 and -2.
Since both eigenvalues are negative, the origin is asymptotically stable equilibrium point.
Matrix has two linearly independent eigenvectors so the origin is a star node.

#5) $\vec{x}' = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \vec{x}$ (node)

Eigenvalues are 1 and 5. The origin is unstable equilibrium point. Since the eigenvalues are real and not equal, the origin is nondegenerate node.

#17) $\vec{x}' = \begin{bmatrix} k & 0 \\ 0 & -1 \end{bmatrix} \vec{x}$

So $\begin{vmatrix} k-\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} = (\lambda - k)(\lambda+1) = 0$

$\lambda_1 = k \quad \lambda_2 = -1$

a) $k < -1$ means the origin is asymptotically stable non-degenerate node.

b) $k = -1$ means origin is asymptotically stable star node.

c) $-1 < k < 0$ means stable non-degenerate node.

d) $k = 0$ means matrix is singular, so origin is not an equilibrium.

e) $k > 0$ means origin is an unstable saddle point.

#14) $x' = xy$

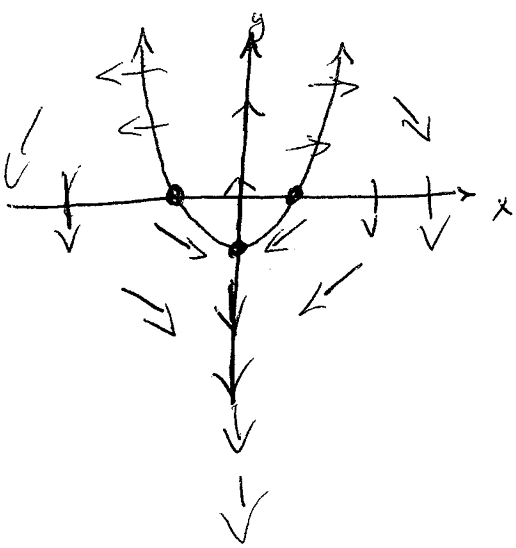
$y' = y - x^2 + 1$

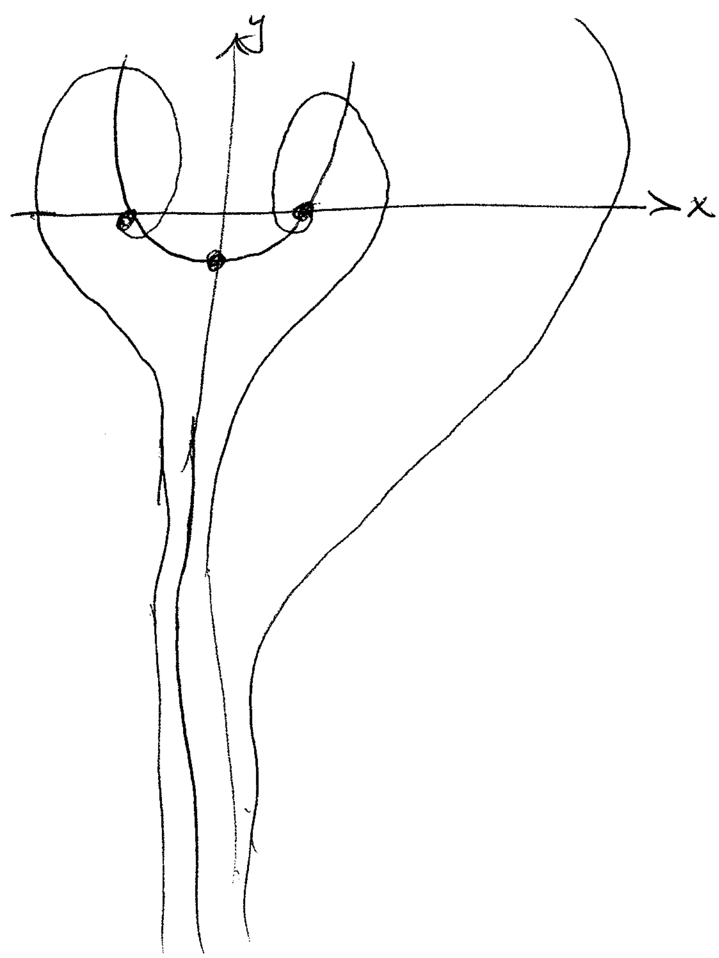
v-null clines: x & y-axes

h-null clines: $y = x^2 + 1$

Equilibria: $(0,-1)$, $(1,0)$, $(1,0)$

All these equilibria are unstable.





#18)

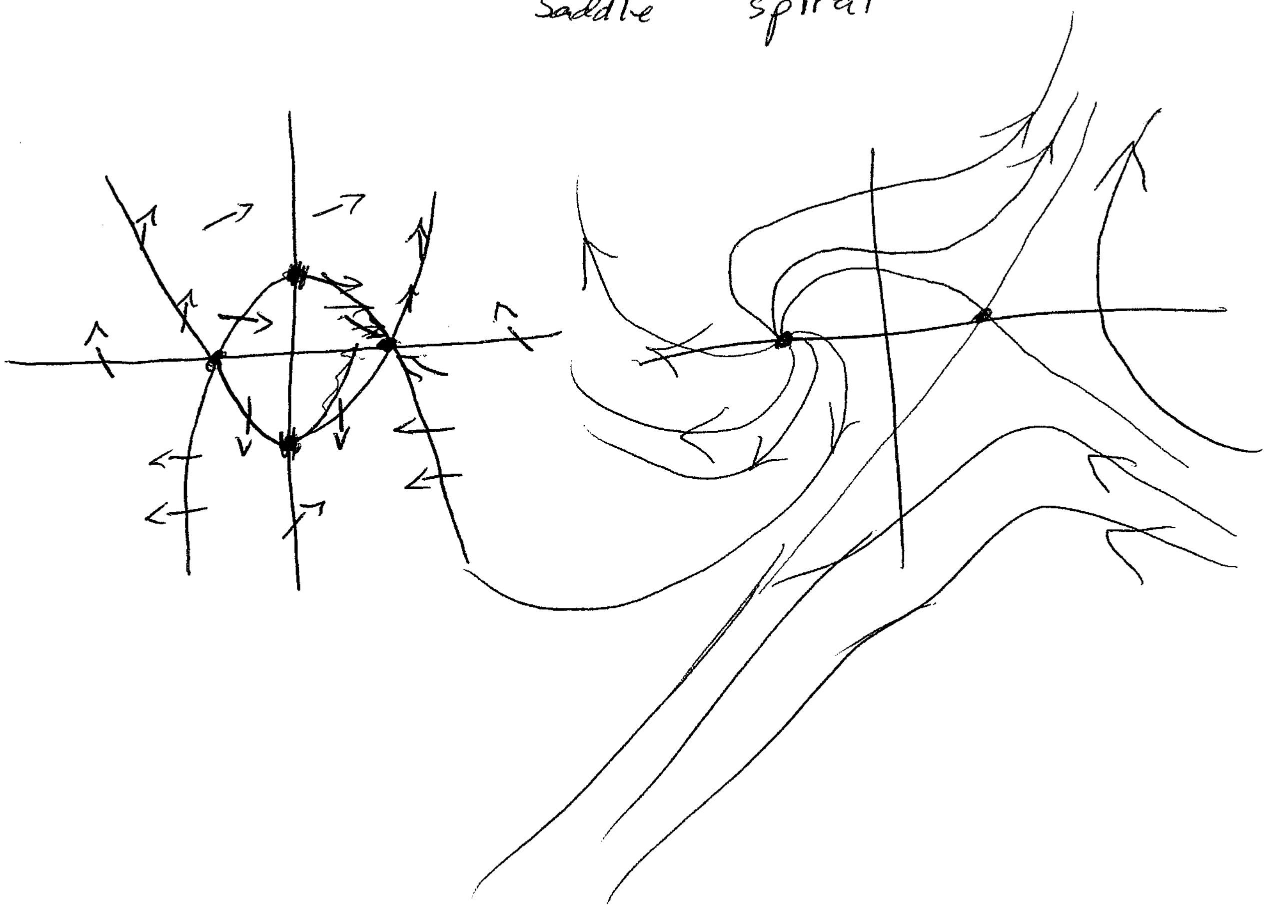
$x' = y - x^2 + 1$ v-nullclines $y = x^2 - 1$

$y' = y + x^2 - 1$ h-nullclines $y = -x^2 + 1$

Equilibria: $(-1, 0)$, $(1, 0)$

$(-1,0)$: unstable saddle

$(1,0)$: unstable spiral



#23) $x'' + (x')^2 + x^2 = 0$

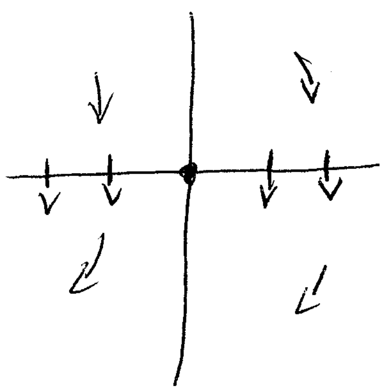
a, b) Let $y = x'$

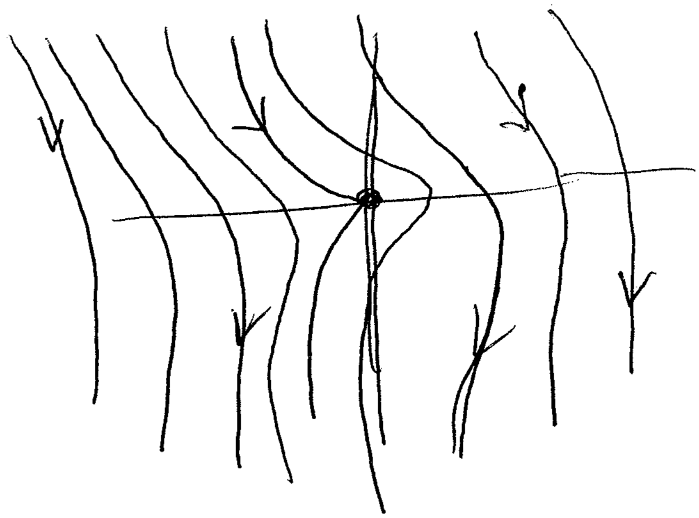
v-nullclines : $y = 0$ x-axis

h-nullclines : $x^2 + y^2 = 0$
origin

$$y' = -x^2 - y^2$$

Equilibrium : $(0,0)$ — unstable





#32) $x' = 2xy$

$y' = y^2 - x^2$

Equilibrium point $(0,0)$

merger of stable and unstable equilibrium.