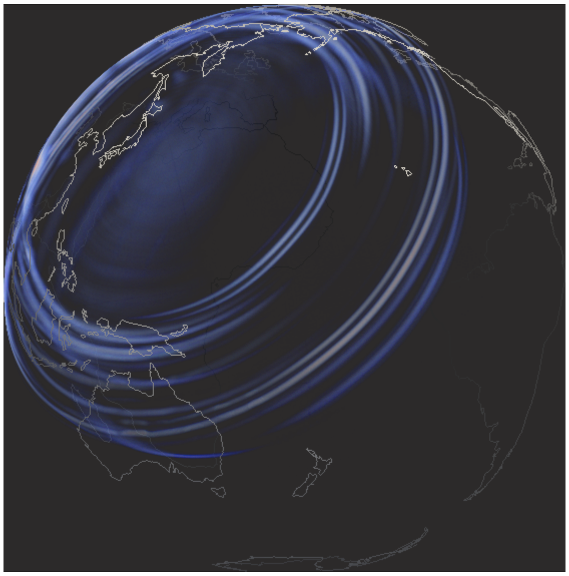
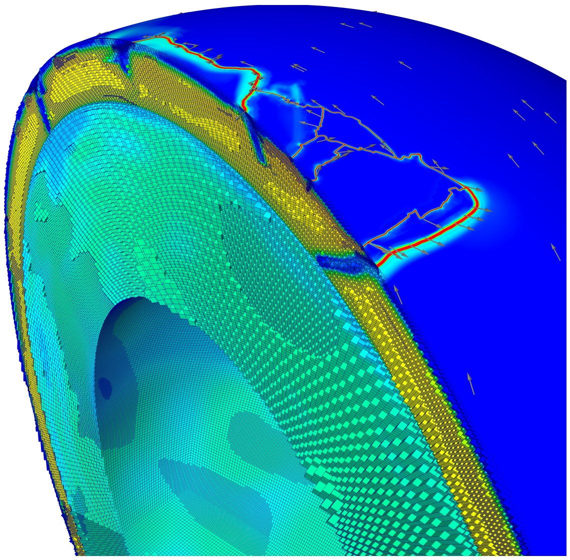
Computational and Variational Methods for Inverse Problems—Spring 2025
Cross-listed as CSE 393P, GEO 391, ME 397, ORI 391Q





Unique numbers: 63175 (CSE), 27790 (GEO), 19725 (ME), 19965 (ORI)

Lectures: Mon/Wed 09:00–11:00, GDC 4.304. While detailed lecture notes will be distributed for each lecture, students are still expected to attend each class.

Office Hours: will be held immediately after class. In addition, feel free to contact me to set up meetings at other times.

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Canvas: We'll be using Canvas (https://utexas.instructure.com/courses/1410692) for distributing handouts, assignments, lecture notes, announcements, etc. Note that I combined all four sections into one listing in Canvas for the sake of convenience. Canvas isn't great as a discussion platform, so we'll also be using Slack for discussion of anything related to the course. You should receive an email invitation to join our Slack workspace, *Inverse Problems Spring 2025*. Students who wish to sit in on the course should register as auditors. If space permits, postdocs are also welcome; send me your EID and I'll add you to the Canvas site and the Slack workspace.

Description: This course provides an introduction to the numerical solution of inverse problems that are governed by systems of partial differential equations (PDEs). The focus of the course is on variational formulations, ill-posedness, regularization, adjoint methods for gradients and Hessians, variational discretization, and efficient large-scale optimization algorithms for inverse problems. Students will develop numerical implementations for model problems using the inverse problems library hIPPYlib (https://hippylib.github.io), which builds on the high-level finite element toolkit FEniCS (https://fenicsproject.org) for discretization and the HPC library PETSc (https://petsc.org) for scalable and efficient linear algebra operations and solvers. These implementations will allow us to study the influence of data noise, regularization, the observation operator, the choice of the parameter field, and the nature of the underlying PDE model on the identifiability of the model parameters, as well as facilitating experimentation with different solution algorithms. The course will also provide a brief introduction to the Bayesian framework and draw connections between the classical and the Bayesian interpretations of inverse problems. Examples will be drawn from different areas of science and engineering, including continuum fluid and solid mechanics, geophysics, and image processing.

Learning outcomes: At the conclusion of this course, students will be able to:

- Understand the nature of ill-posedness of inverse problems;
- Invoke appropriate regularization methods to make inverse problems well-posed;
- Employ state-of-the-art Newton optimization methods to solve regularized inverse problems;
- Use variational methods for first and second order sensitivity analysis of PDE-governed inverse problems;
- Implement inverse solvers for a variety of PDE-governed inverse problems in the hIPPYlib/FEniCS framework; and
- Understand the relationship between classical and Bayesian inverse problems.

Prerequisites: Graduate standing or consent of instructor, and a background in numerical linear algebra and partial differential equations. Familiarity with Python and Jupyter notebooks is expected. Prior exposure to nonlinear optimization is desirable, but not essential. The required mathematical background will be covered when needed—albeit quickly. A mathematically mature student will be able to acquire from the lectures the necessary mathematical and computational background. If in doubt, contact me.

Required work and grading policy: Four or five (rather lengthy) assignments involving a mix of theory, implementation, computational experiments, and interpretation. No midterm or final exam. Your final grade will be based on an average of the assignments, equally weighted. Letter grades will be assigned as follows:

95–100: A+
90–95: A
85–90: A-
80–85: B+
75–80: B
70–75: B-
65–70: C+
60–65: C
55–60: C-
Below 55: D

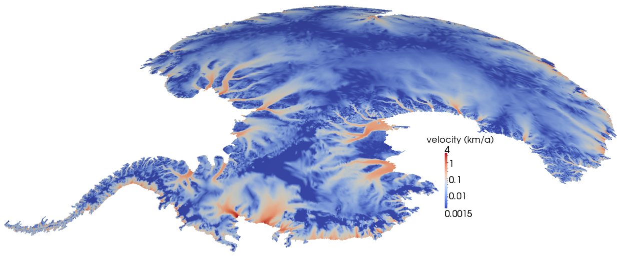
Students taking the class on a Credit/No-Credit basis must maintain a 60% or higher average on the assignments to receive credit.

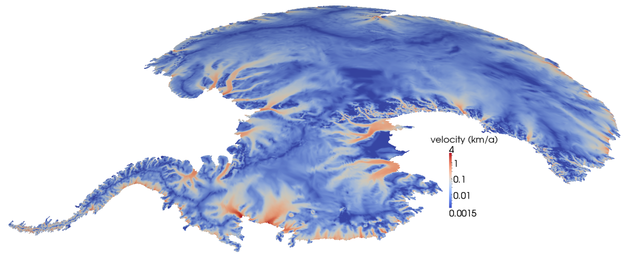
Collaboration policy: Students are encouraged to discuss among themselves the course material and assignments. However, all turned-in material must be the work of the individual student. Students who violate University rules on academic misconduct are subject to the student conduct process and potential disciplinary action. A student found responsible for academic misconduct may be assigned both a status sanction and a grade impact for the course. The grade impact could range from a zero on the assignment in question up to a failing grade in the course. A status sanction can range from probation, deferred suspension and/or dismissal from the University. To learn more about academic integrity standards, tips for avoiding a potential academic misconduct violation, and the overall conduct process, please visit the Student Conduct and Academic Integrity website at http://deanofstudents.utexas.edu/conduct.

Class recordings: Class recordings are reserved only for students in this class for educational purposes and are protected under FERPA. The recordings should not be shared outside the class in any form.

Sharing of course materials: No materials used in this class, including, but not limited to, lecture notes, handouts, videos, assignments, and in-class materials, may be shared online or with anyone outside of the class unless you have my explicit, written permission.

Accessibility: The university is committed to creating an accessible and inclusive learning environment consistent with university policy and federal and state law. Please let me know if you experience any barriers to learning so I can work with you to ensure you have equal opportunity to participate fully in this course. If you are a student with a disability, or think you may have a disability, and need accommodations, please contact Disability and Access (D&A). Please refer to D&A's website for contact and more information: https://disability.utexas.edu/. If you are already registered with D&A, please present your Accommodation Letter to me as early as possible in the semester so we can discuss your approved accommodations and needs in this course.

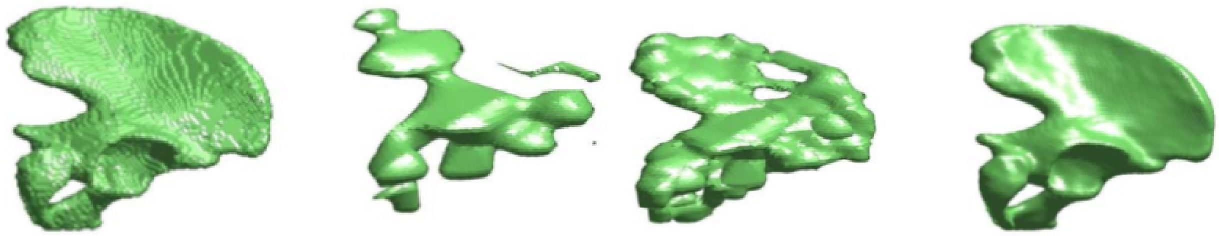




Course Topics

The following is my expectation for the number of lectures devoted to each topic. The pace may need to change to enhance class learning.

- Introduction and examples of inverse problems with PDEs (1 lecture)
- Ill-posed problems and regularization (5 lectures)
  - Theoretical aspects
  - Different regularization methods
  - Choice of regularization parameter
  - Structure and properties of the Hessian
- Numerical optimization methods (6 lectures)
  - Optimality criteria
  - Line search globalization
  - Steepest descent
  - Newton method
  - Gauss-Newton method
  - Inexact Newton-conjugate gradient method
  - Automatic differentiation
- Variational methods, weak forms, Galerkin methods, FEniCS (3 lectures)
- Sensitivity analysis (7 lectures)
  - Direct and adjoint
  - Steady and unsteady problems
  - Discrete vs. continuous
  - Linear and nonlinear PDEs
  - Distributed, boundary, and finite-dimensional parameters and data
- Combining discretizations, optimizers, and sensitivity analysis to solve PDE-constrained inverse problems with hIPPYlib (2 lectures)
- Bayesian approach to inverse problems and optimal experimental design (2 lectures)



Books on Inverse Problems

Required textbook

This course does not have a required textbook—instead, I will distribute my own lecture notes. In addition, a 60-page condensed version of the course can be found in Part 2 of our *Acta Numerica* paper:

- O. Ghattas and K. Willcox, Learning physics-based models from data: perspectives from inverse problems and model reduction, *Acta Numerica*, 30:445–554, 2021. https://doi.org/10.1017/S0962492921000064

Recommended books on theory and computational methods for inverse problems:

- H.T. Banks and K. Kunisch, *Estimation Techniques for Distributed Parameter Systems*, Systems & Control: Foundations & Applications, Birkhäuser, 1989.
- Heinz Engl, Michael Hanke, and Andreas Neubauer, *Regularization of Inverse Problems*, Dordrecht, 2nd edition, 1996.
- Curtis R. Vogel, *Computational Methods for Inverse Problems*, SIAM, 2002.
- A. Kirsch, *An Introduction to the Mathematical Theory of Inverse Problems*, second edition, Springer, 2011.
- Guy Chavent, *Nonlinear Least Squares for Inverse Problems*, Springer, 2009.
- Per Christian Hansen, *Discrete Inverse Problems: Insight and Algorithms*, SIAM, 2010.
- Jennifer Mueller and Samuli Siltanen, *Linear and Nonlinear Inverse Problems with Practical Applications*, SIAM, 2012.
- M. Asch, M. Bocquet and M. Nodet, *Data Assimilation: Methods, Algorithms, and Applications*, SIAM, 2016.

Recommended books on numerical optimization:

- Jorge Nocedal and Stephen J. Wright, *Numerical Optimization*, Springer-Verlag, 1999.
- C. Tim Kelley, *Iterative Methods of Optimization*, SIAM, 1999.

Recommended books on optimization of systems governed by PDEs:

- Max D. Gunzburger, *Perspectives in Flow Control and Optimization*, SIAM, 2003.
- M. Hinze, R. Pinnau, M. Ulbich, and S. Ulbrich, *Optimization with PDE constraints*, Springer, 2009.

- Fredi Tröltzsch, *Optimal Control of Partial Differential Equations: Theory, Methods and Applications*, Graduate Studies in Mathematics Vol. 112, AMS, 2010.
- Alfio Borzì and Volker Schulz, *Computational Optimization of Systems Governed by Partial Differential Equations*, SIAM, 2012.

Recommended books on uncertainty quantification in inverse problems:

- Albert Tarantola, *Inverse Problem Theory and Methods for Model Parameter Estimation*, SIAM, 2005.
- Jari Kaipio and Erkki Somersalo, *Statistical and Computational Inverse Problems*, Springer, 2005.
- Ralph C. Smith, *Uncertainty Quantification: Theory, Implementation, and Applications*, SIAM, 2013.
- Tim J. Sullivan, *Introduction to Uncertainty Quantification*, Springer, 2015.
- Luis Tenorio, *An Introduction to Data Analysis and Uncertainty Quantification for Inverse Problems*, SIAM, 2017.
- Johnathan Bardsley, *Computational Uncertainty Quantification for Inverse Problems*, SIAM, 2018.