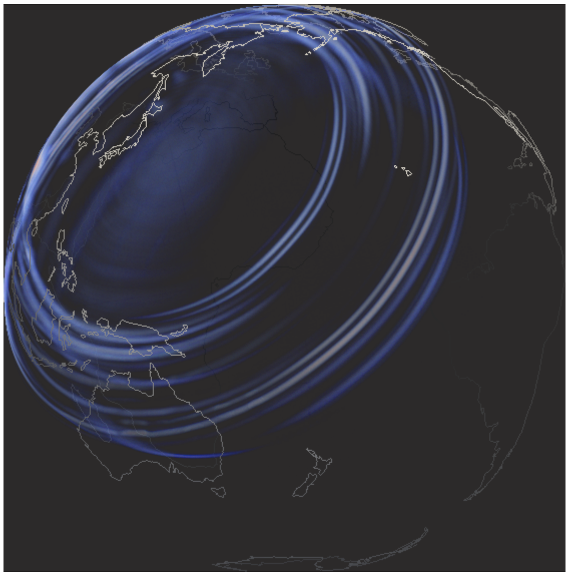
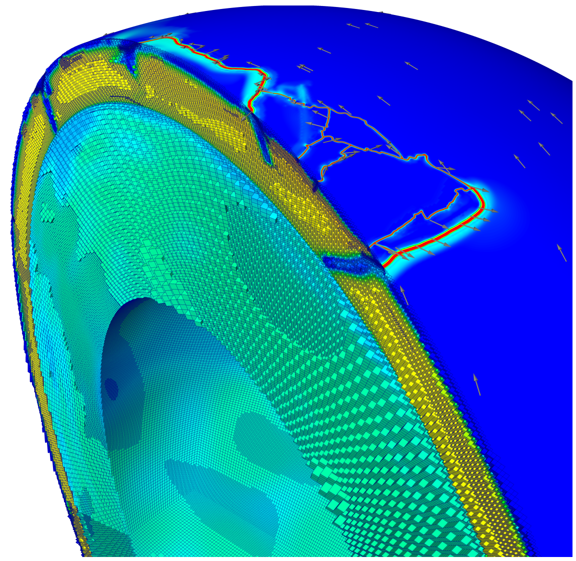
Spring 2023
Computational and Variational Methods for Inverse Problems
Cross-listed as CSE 393P, GEO 391, ME 397, ORI 397





Unique numbers: 63014 (CSE), 27864 (GEO), 19389 (ME), 19648 (ORI)

Lectures: Mon/Wed 08:00–10:00, GDC 4.302. Office hours will be held immediately after class. In addition, feel free to contact me to set up meetings at other times.

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Canvas: We'll be using Canvas (https://utexas.instructure.com/courses/1355560) for distributing handouts, assignments, lecture notes, announcements, etc. Note that I combined all four sections into one listing in Canvas for the sake of convenience. Canvas isn't great as a discussion platform, so we'll also be using Slack for discussion. For those students/postdocs who are not registered and wish to sit in on the course, send me your EID and I'll add you to the Canvas site and the Slack workspace.

Description: This course provides an introduction to the numerical solution of inverse problems that are governed by systems of partial differential equations (PDEs). The focus of the course is on variational formulations, ill-posedness, regularization, adjoint methods for gradients and

Hessians, variational discretization, and efficient large-scale optimization algorithms for inverse problems. Students will develop numerical implementations for model problems using the inverse problems library hIPPYlib (https://hippylib.github.io), which builds on the high-level finite element toolkit FEniCS (https://fenicsproject.org) for discretization and the HPC library PETSc (https://www.mcs.anl.gov/petsc) for scalable and efficient linear algebra operations and solvers. These implementations will allow us to study the influence of data noise, regularization, the observation operator, the choice of the parameter field, and the nature of the underlying PDE model on the identifiability of the model parameters, as well as facilitating experimentation with different solution algorithms. The course will also provide a brief introduction to the Bayesian framework and draw connections between the classical and the Bayesian interpretations of inverse problems. Examples will be drawn from different areas of science and engineering, including continuum fluid and solid mechanics, geophysics, and image processing.

Prerequisites: Graduate standing or consent of instructor. A background in numerical linear algebra, partial differential equations, and nonlinear optimization is desirable. However, the required mathematical background will be covered when needed—albeit quickly. A mathematically mature student will be able to acquire from the lectures the necessary mathematical and computational background. If in doubt, contact me.

Required work: Five or six assignments involving a mix of theory, implementation, and computational experiments. Your final grade will be based on an average of the assignments, equally weighted. Letter grades will be assigned as follows:

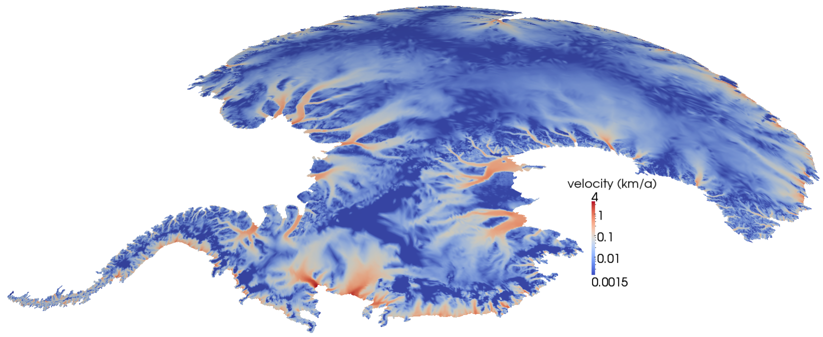
95–100: A+
90–95: A
85–90: A-
80–85: B+
75–80: B
70–75: B-
65–70: C+
60–65: C
55–60: C-
Below 55: D

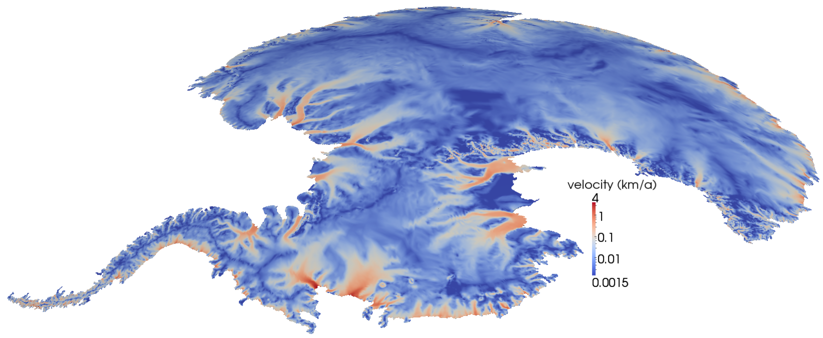
Students taking the class on a Credit/No-Credit basis must maintain a 60% or higher average on the assignments to receive credit.

Collaboration policy: Students are encouraged to discuss among themselves the course material and assignments. However, all turned-in material must be the work of the individual student.

Class recordings: Class recordings are reserved only for students in this course for educational purposes and are protected under FERPA. The recordings should not be shared outside the class in any form.

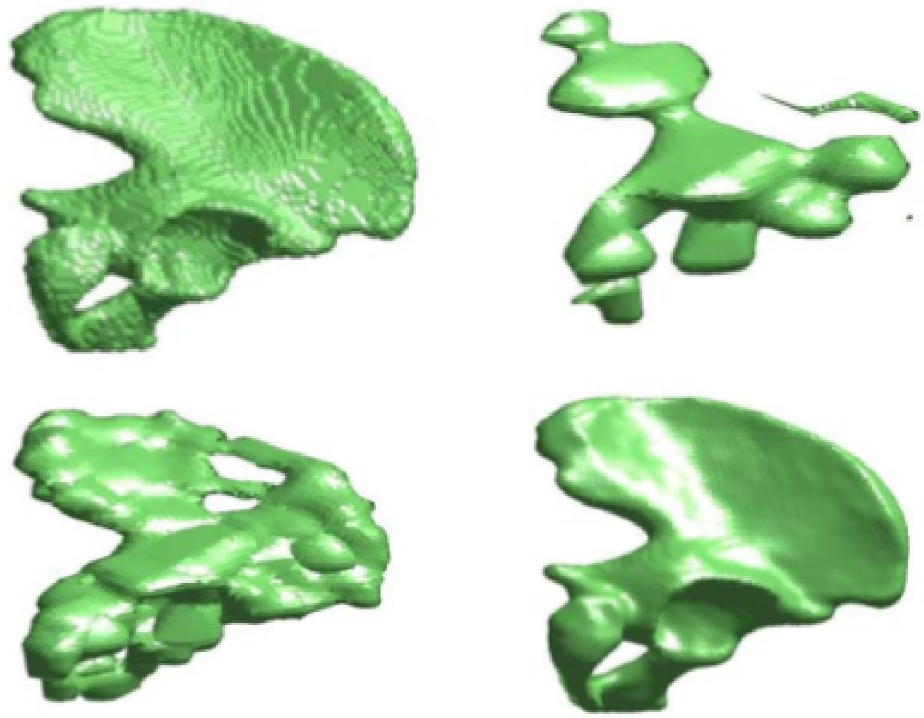
Sharing of course materials: No materials used in this class, including, but not limited to, lecture notes, handouts, videos, assignments, and in-class materials, may be shared online or with anyone outside of the class unless you have my explicit, written permission.





Course Topics

- Introduction and examples of inverse problems with PDEs
- Ill-posed problems and regularization
  - Theoretical aspects
  - Different regularization methods
  - Choice of regularization parameter
- Numerical optimization methods
  - Line search globalization
  - Steepest descent
  - Newton method
  - Gauss-Newton method
  - Inexact Newton-conjugate gradient method
- Variational methods, weak forms, Galerkin methods
- Sensitivity analysis
  - Direct and adjoint
  - Steady and unsteady problems
  - Discrete vs. continuous
  - Linear and nonlinear PDEs
  - Distributed, boundary, and finite-dimensional parameters and data
- Bayesian approach to inverse problems



Books on Inverse Problems

Textbook

This course does not have a textbook—instead, I will distribute my own lecture notes. In addition, a 60-page summary (more or less) of the entire course can be found in Part 2 of our *Acta Numerica* paper:

- O. Ghattas and K. Willcox, Learning physics-based models from data: perspectives from inverse problems and model reduction, *Acta Numerica*, 30:445–554, 2021. https://doi.org/10.1017/S0962492921000064

Recommended books on theory and computational methods for inverse problems:

- H.T. Banks and K. Kunisch, *Estimation Techniques for Distributed Parameter Systems*, Systems & Control: Foundations & Applications, Birkhäuser, 1989.
- Heinz Engl, Michael Hanke, and Andreas Neubauer, *Regularization of Inverse Problems*, Dordrecht, 2nd edition, 1996.
- Curtis R. Vogel, *Computational Methods for Inverse Problems*, SIAM, 2002.
- A. Kirsch, *An Introduction to the Mathematical Theory of Inverse Problems*, second edition, Springer, 2011.
- Guy Chavent, *Nonlinear Least Squares for Inverse Problems*, Springer, 2009.
- Per Christian Hansen, *Discrete Inverse Problems: Insight and Algorithms*, SIAM, 2010.
- Jennifer Mueller and Samuli Siltanen, *Linear and Nonlinear Inverse Problems with Practical Applications*, SIAM, 2012.

- M. Asch, M. Bocquet and M. Nodet, *Data Assimilation: Methods, Algorithms, and Applications*, SIAM, 2016.

Recommended books on numerical optimization:

- Jorge Nocedal and Stephen J. Wright, *Numerical Optimization*, Springer-Verlag, 1999.
- C. Tim Kelley, *Iterative Methods of Optimization*, SIAM, 1999.

Recommended books on optimization of systems governed by PDEs:

- Max D. Gunzburger, *Perspectives in Flow Control and Optimization*, SIAM, 2003.
- M. Hinze, R. Pinnau, M. Ulbich, and S. Ulbrich, *Optimization with PDE constraints*, Springer, 2009.
- Fredi Tröltzsch, *Optimal Control of Partial Differential Equations: Theory, Methods and Applications*, Graduate Studies in Mathematics Vol. 112, AMS, 2010.
- Alfio Borzì and Volker Schulz, *Computational Optimization of Systems Governed by Partial Differential Equations*, SIAM, 2012.

Recommended books on uncertainty quantification in inverse problems:

- Albert Tarantola, *Inverse Problem Theory and Methods for Model Parameter Estimation*, SIAM, 2005.
- Jari Kaipio and Erkki Somersalo, *Statistical and Computational Inverse Problems*, Springer, 2005.
- Ralph C. Smith, *Uncertainty Quantification: Theory, Implementation, and Applications*, SIAM, 2013.
- Tim J. Sullivan, *Introduction to Uncertainty Quantification*, Springer, 2015.
- Luis Tenorio, *An Introduction to Data Analysis and Uncertainty Quantification for Inverse Problems*, SIAM, 2017.
- Johnathan Bardsley, *Computational Uncertainty Quantification for Inverse Problems*, SIAM, 2018.