ASE 211 Homework 8 Solution

1. Given data (x_i, y_i), $i = 1, \ldots, n$, write two matlab m-files, the first of which generates a least squares polynomial fit, for any polynomial of degree $q \leq n$, and a second which evaluates the polynomial at any given point.

The first m-file should have the following structure:

```
function [alpha]=ls(x,y,n,q)
%
%  generates a least squares polynomial of specified degree q using
%  data x(i),y(i),i=1,...,n
%
%  returns the coefficients alpha(1), alpha(2), ..., alpha(q+1)
%  of the polynomial
%
A=zeros(q+1,q+1);
b=zeros(q+1,1);
for i=1:q+1
  for k=1:n
     b(i)=b(i)+y(k)*x(k)^(q+1-i);
     for j=1:q+1
        A(i,j)=A(i,j)+x(k)^(q+1-j)*x(k)^(q+1-i);
     end
  end
end
alpha=A\b;
```

The second m-file should have the following structure:

```
function yy=evalls(alpha,q,xx)
%
%  evaluates the least squares polynomial of degree q at the point
%  xx, and returns the value in yy
%
%  the polynomial has the form
%     yy = alpha(1)*xx^q+alpha(2)*xx^(q-1)+...+alpha(q)*xx+alpha(q+1)
%
yy=0;
for i=1:q+1
```

```
 yy=yy+alpha(i)*xx^(q-i+1);
end
```

2. Test your m-files on the data given in the table below for polynomials of degree $q = 1, \ldots, 7$.

Plot your least squares polynomials versus the actual data as given in the sample figure below. Also generate and plot a cubic spline which interpolates the data, and compare to your least squares solutions. Which of the functions do you think best represents the data (justify your answer)? Hand in all plots and m-files. You do not have to hand in a diary.

Here is the data:

x	y
0	3.3
.5	2.1
1.5	-.5
1.6	1.2
2.8	6.9
3.1	8.1
4.0	7.9
5.5	4.3
6.2	1.1
6.5	-.1
7.0	-3.2
8.2	-4.5
9.6	-2.4
9.8	-2.4
11.1	1.9
12.5	2.6
13.0	4.7

Discussion: The least squares solutions for $q = 6$ and $q = 7$ model the data pretty well without any nonphysical oscillation. These solutions look better than the spline solution, which oscillates quite a bit.

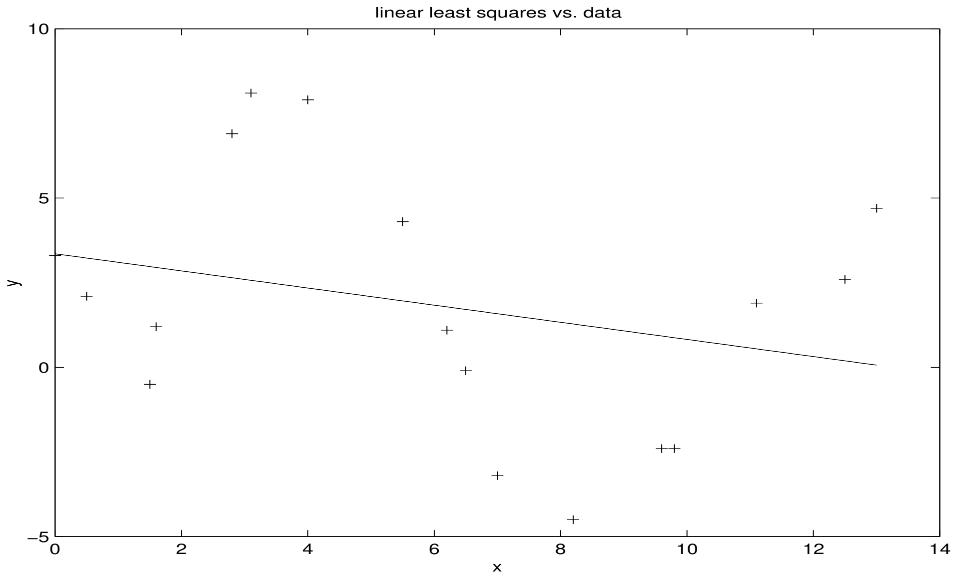

Figure 1: Least squares solution for $q = 1$

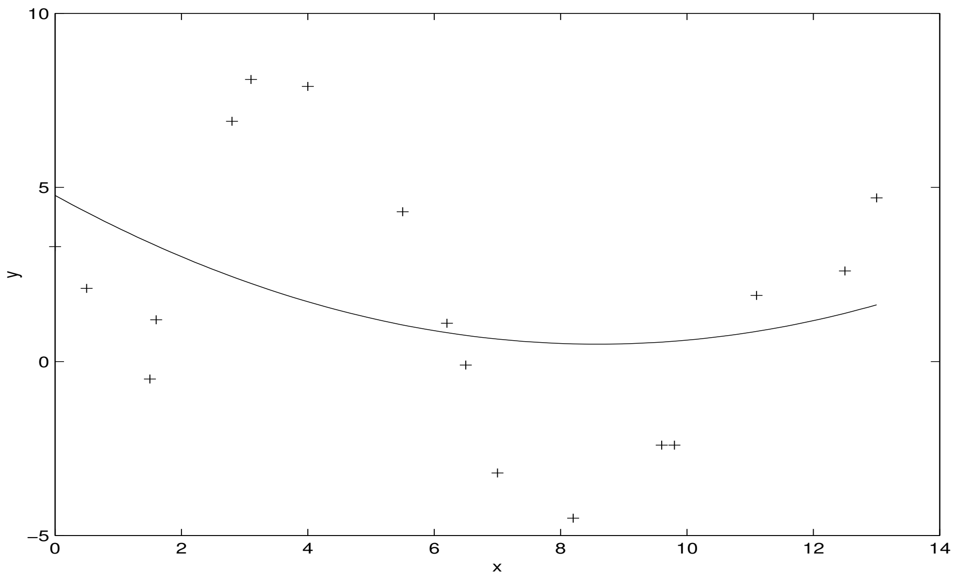

Figure 2: Least squares solution for $q = 2$

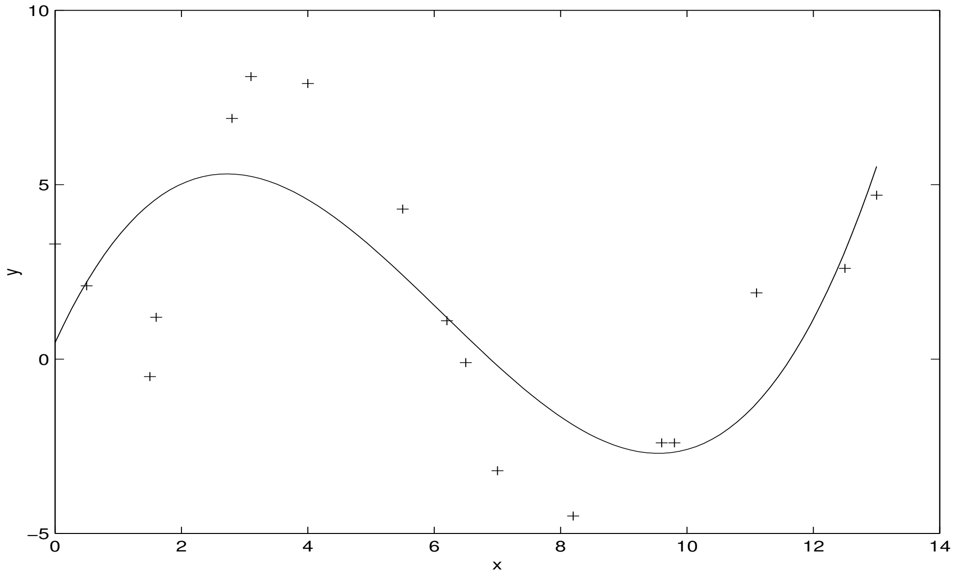

Figure 3: Least squares solution for $q = 3$

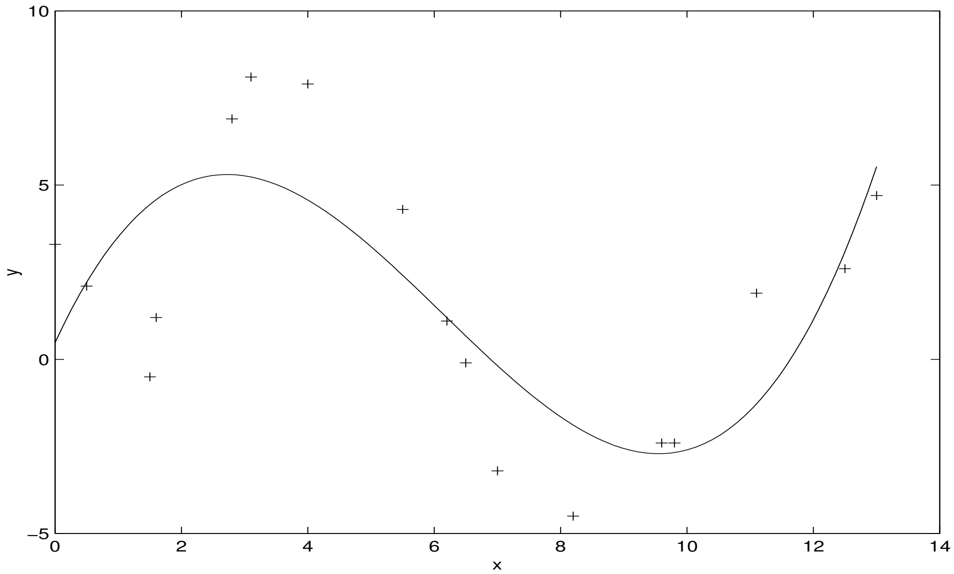

Figure 4: Least squares solution for $q = 4$

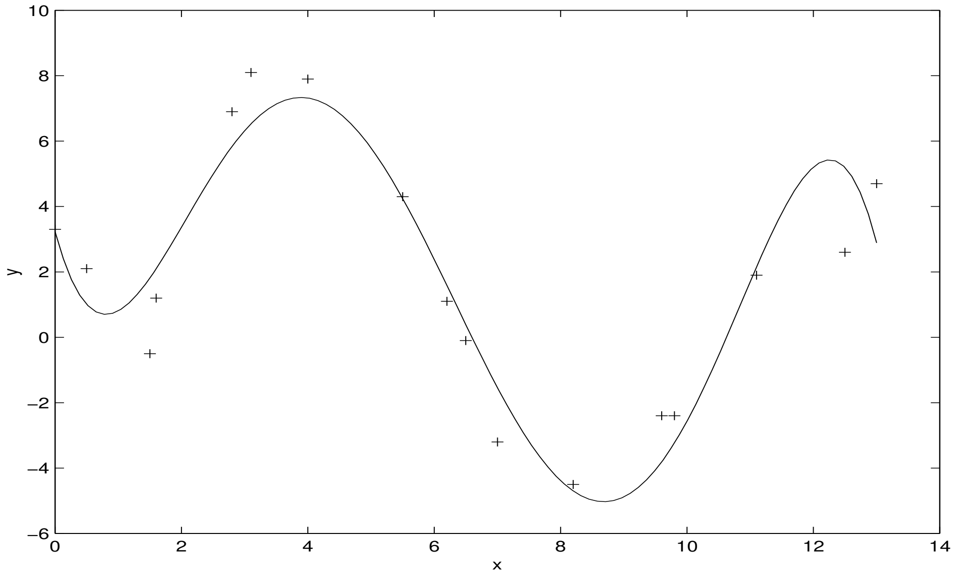

Figure 5: Least squares solution for $q = 5$

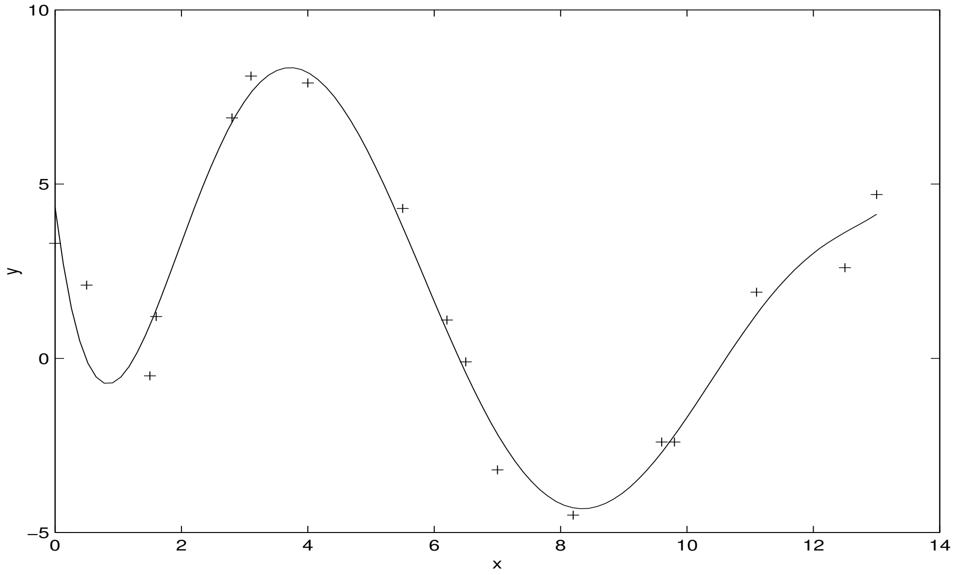

Figure 6: Least squares solution for $q = 6$

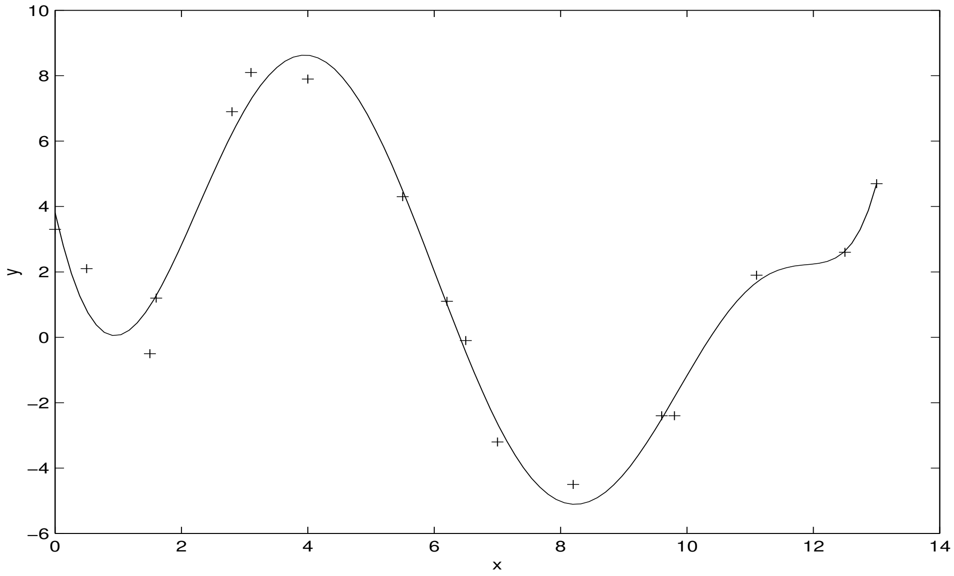

Figure 7: Least squares solution for $q = 7$

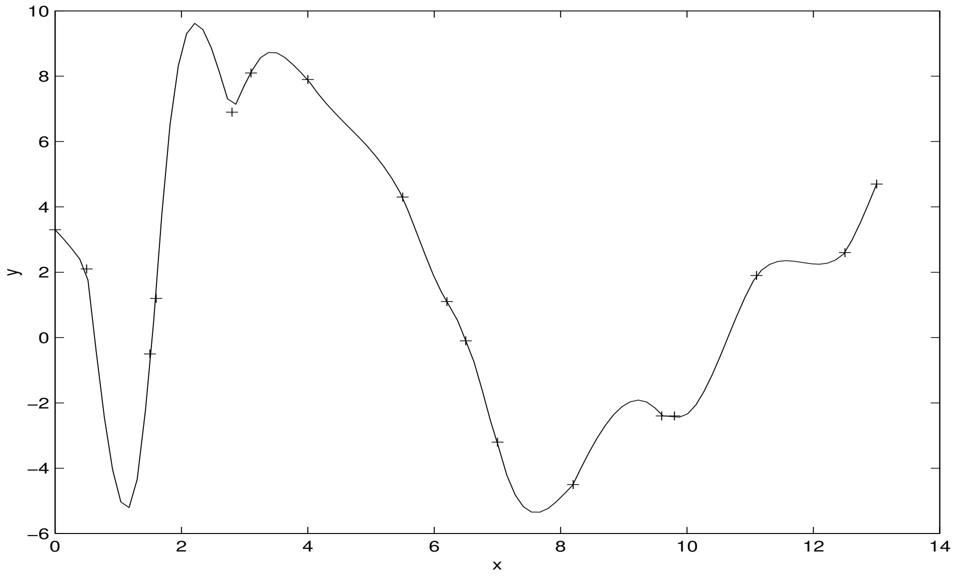

Figure 8: Cubic spline solution